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THE GALACTIC STRUCTURE IN TAURUS
I. SURFACE DISTRIBUTION OF STARS

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ABSTRACT

In order to study the structural features of the galaxy toward the anti-center, general star counts to photographic magnitude 15 have been made in an area of 600 square degrees in Taurus between α 2^h40^m and 5^h40^m and δ $+20°$ to $+35°$. The magnitude scale corresponds essentially to the International Photographic Scale, although there may be a systematic difference amounting to possibly 0.2 mag. for the brightest stars.

The surface distribution of stars in arbitrarily selected regions of little or no obscuration is given in Table 4 and is compared with similar data by others. It is found that these areas exhibit an excess of 0.2 in log $N(m)$ over the data published by van Rhijn and by Seares and Joyner. An examination of the plates shows that this excess is undoubtedly due to the obscuring nebulosity overlying the Selected Areas on which previous stellar distributions in this region have been based.

With the mean values of log $N(m)$ in the clear areas as a basis, the amount of absorption of photographic light at magnitude 15 has been determined for small subareas, the distribution being shown by a chart and by a contour map (Figs. 1 and 3).

The total absorption at the fifteenth magnitude in the condensations of the Taurus region reaches values as high as 3.5–3.7 mag. Over much of the region the obscuration amounts to 1.5–2.5 mag.

The stellar space density in Taurus and a discussion of the dark nebulae will be presented in a succeeding paper.

Investigations of galactic structure during the last twenty or thirty years have shown conclusively that the complexity of interstellar absorbing clouds renders discussion of mean stellar distributions in low galactic latitudes rather useless. It has become apparent that detailed analyses of individual regions are necessary if an accurate picture of the structure of the Milky Way is to be obtained. Numerous investigations based on this theme have been

inaugurated at the Harvard College Observatory,[1] and the present paper gives the first results of a study of the region in Taurus which is heavily obscured by dark nebulosity.

Many years ago Barnard[2] called attention to the nebulosity around the Pleiades and to the dark lanes over the whole Taurus region. Since then several detailed studies of particular parts of this nebulous groundwork have been made, with results which are in good agreement as to the distance and absorbing power of the nebula. A summary of this work has been given recently by Bok,[3] and hence it will suffice to mention here only the general result of the past research.

Studies by general star counts and by analysis of spectral type distributions have shown that a large region of the sky (some 600 square degrees), covering much of the constellations of Auriga, Taurus, and Orion, is obscured by dark material setting in at a distance of 100–150 parsecs and absorbing, on the average, one magnitude in photographic light. Numerous condensations of this material show absorptions as high as 2.5–4 mag.; these constitute the dark lanes and holes referred to by Barnard.

In contrast to the previous investigations covering small regions, the present analysis has been extended by general star counts over the entire region of the sky from α 2^h40^m to 5^h40^m and from $\delta + 20°$ to $+35°$—an area of approximately 600 square degrees.

THE OBSERVATIONAL MATERIAL

Thirty-two plates having images of high quality were taken on photometric nights with the 4-inch Ross-Lundin patrol camera at the Oak Ridge Station of the Harvard Observatory. In an hour's exposure time with Cramer Hi-Speed Special plates the average limiting magnitude for the counts was 15.

[1] The present paper and those dealing with the analysis of the material on stellar distribution in Taurus, which are to follow soon, together form Part III of the series "Investigations of Galactic Structure," which is being published as a result of co-operative research carried on between the Harvard College Observatory and astronomers elsewhere. See Bok, *The Distribution of the Stars in Space* ("Astrophysical Monograph"), p. 58, 1937, for details concerning this series.

[2] *Ap. J.*, **25**, 218, 1907.

[3] *The Distribution of the Stars in Space* ("Astrophysical Monograph"), p. 77, 1937.

These plates were spaced in right ascension and declination so as to cover, with considerable overlapping, the region of the sky indicated in the previous section. Uniformity of sky conditions and of exposure time was made a requisite for the entire series.

By a method described previously by F. D. Miller,[4] star counts to successive photographic magnitude limits from 8 to 15 were made on an average area of 39 square degrees per plate. The total number of stars counted, including the overlapping counts, was approximately 300,000. Counts were not made within 0.5 mag. of the limiting magnitude of the plate because of the uncertainties involved. Sequences of stars having known magnitudes were established on each plate for the purpose of calibrating the counts. The final product of counting is represented by values of log $N(m)$, where $N(m)$ is the number of stars per square degree brighter than photographic magnitude m. These, of course, may be converted into values of $A(m)$, the number of stars between apparent magnitudes $m - \frac{1}{2}$ and $m + \frac{1}{2}$, for purposes of space-density analysis. An attempt has been made in this way to present both a general and a detailed picture of this portion of the Milky Way and its environs on as homogeneous a magnitude scale as possible.

THE MAGNITUDE SYSTEM

A most important requirement for any study of galactic structure based upon star counts is homogeneity of the magnitude system over the entire counted area. Furthermore, it is desirable that counts such as the present should agree in scale and zero point as closely as possible with the International System of photographic magnitudes.

The magnitudes published by Seares, Kapteyn, and van Rhijn in the *Mount Wilson Catalogue of Selected Areas 1–139* have been adopted for the region in Taurus as a basis for the counts. Selected Areas 22, 23, 24, 47, 48, and 49 fall within the area counted, and, of these, 47, 48, and 49 lie within the Taurus nebulosity in the central part of the region. Accordingly, the photographic magnitudes from $m = 12$ to $m = 15$ have been taken directly from the *Mount Wilson Catalogue* for these three areas and serve as primary calibration standards for the fainter stars.

[4] *Harvard Ann.*, **105**, 298, 1937.

Until the *Bergedorfer Spektral-Durchmusterung*, which contains accurately determined photographic magnitudes in the Selected Areas for stars between $m = 9$ and $m = 13$, is complete for the zone at $+30°$ declination, the magnitude data for the brighter range included in the present counts will be somewhat uncertain. For the time being, polar comparison plates taken at Oak Ridge, supplemented by similar plates taken at the Warner and Swasey Observatory, have been used to provide magnitude sequences for the brighter stars.

Some of the Harvard plates, taken with the 3-inch Ross-Fecker (RH) lens, contained two exposures of six minutes each, one on the pole and the other on a Selected Area. Others contained interlocking exposures of the Selected Areas so that a complete network of comparisons between the pole and the Selected Areas could be established. In general, however, only one polar plate was available for each Selected Area. The exposures were arranged so that the polar sequence stars and the Area stars were intermingled on the plate. While the quality of images on the RH plates was not as good as on the RL plates used for counting, still there was sufficient comparability between the polar images and the Area images to insure accurate measures.

By means of the same artificial scale of stellar images used in counting, magnitudes were determined from the polar comparison plates for eight or ten stars of each Selected Area in the magnitude range 7 to 12. Interlocking Selected Area plates were compared in a similar way. Two series of estimates made six months apart were used to obtain some knowledge of the uncertainty in the process, and the average probable error of a single magnitude determination was found to be ± 0.13 mag. This result was derived from the polar comparison plates only.

Added data on the magnitudes of the bright stars for Selected Areas 47, 48, and 49 were obtained from a series of polar comparison plates taken at the Warner and Swasey Observatory. The magnitudes, based as in the RH series on the International System,[5] were obtained by eye estimates, each star being measured on two plates. The average probable error for a single observation was found to be ± 0.09 mag.

[5] *Trans. I.A.U.*, pp. 71 and 72, 1922.

The three critical areas from which were obtained sequences suitable for calibrating the counts (Selected Areas 47, 48, and 49) contain, finally, twenty-nine bright stars for which probable errors in the adopted magnitudes average ± 0.05 mag. An intercomparison between the plates taken at Oak Ridge and those taken at Cleveland indicated a possible zero-point difference of 0.07 mag., in the sense that the RH series yielded magnitudes slightly too faint.

A lack of bright stars for Selected Areas 47, 48, and 49 in the *Mount Wilson Catalogue* precludes any significant comparison with the scale and zero point of Seares's magnitudes. Other evidence, however, in the form of star counts based upon the sequences indicates no serious departure from the International System of photographic magnitudes. An intercomparison, on the interlocking plates, of the magnitudes adopted for Selected Areas 47, 48, and 49 from the polar plates shows no systematic tendencies either in scale or in zero point.

Auxiliary sequences established on the overlapping portions of the RL plates used for counting permitted the transfer of the Selected Area standards from plate to plate, so that finally a sequence of known magnitudes was available in each counted area.

The homogeneity for the brighter end ($m < 13$) of the calibration sequences has been tested in four ways: (*a*) by the internal agreement of the magnitudes of a sequence determined through two or more independent transfer paths; (*b*) by comparison with sequences established in the same region by Dr. C. H. Payne-Gaposchkin and Miss Wright;[6] (*c*) by means of the short-exposure polar comparison plates taken at Cleveland; and (*d*) by a comparison of star counts made on overlapping portions of adjacent plates.

a) For ten of the sequences used in calibrating the counts, magnitudes were determined by transferring sequences from two or more adjacent plates independently. A comparison of the magnitudes obtained from one adjacent plate with those obtained for the same stars from another adjacent plate indicated an average systematic difference of 0.08 mag. and a probable error for a single observation of ± 0.13 mag., including the systematic effect. From fifteen to twenty-four stars per sequence were used.

[6] *Harvard Ann.*, **89**, No. 4, 1934.

b), *c*) A comparison of the adopted system of magnitudes with those determined for the *Henry Draper Extension*,[6] as well as with magnitudes determined by direct polar comparison, is exhibited by Table 1. There is little evidence of scale difference between the systems, but a zero-point difference of 0.2 mag. appears to be present.

The reality of such a zero-point difference becomes somewhat doubtful when one investigates the internal structure of the systems with which the present magnitudes are compared. Variations of three- or four-tenths of a magnitude have been found among the

TABLE 1

MAGNITUDE AGREEMENT WITH HDE AND DIRECT POLAR COMPARISONS

Mag.	No. of Stars	RL *minus* HDE	No. of Stars	RL *minus* Polar Sequence
7.0– 7.99.....	16	-0^m17	18	-0^m07
8.0– 8.99.....	21	.17	23	.23
9.0– 9.99.....	22	.02	31	.19
10.0–10.99.....	29	.24	37	.22
11.0–11.99.....	25	.39	7	.25
12.0–12.99.....	11	.28	.....	.....
All.....	124	−0.21	116	−0.19

zero points of individual HDE sequences, while the elongation of images available on the polar comparison plates for the Area stars as compared with the polar images would most certainly lead to systematic error.

In view of the uncertainties mentioned and pending the accumulation of more data, the magnitudes as originally determined have been adopted without revision.

d) The internal consistency of the magnitude system over the area covered by the counts has been tested by comparing the counted numbers on overlapping portions of adjacent plates. Since no two plates were calibrated from the same sequence, any marked internal discrepancy in the magnitude standards should be indicated at once. Twenty-five pairs of overlapping areas, averaging 12.6 square degrees each, yielded the probable errors shown in Table 2. Accidental

and systematic errors in counting are of course included in these values.

In view of the fact that systematic and accidental counting errors have been found to be less than ±0.05 (p.e.) in log $N(m)$ and ±0.15 in magnitude, it is apparent from Table 2 that the internal consist-

TABLE 2

PROBABLE ERRORS FROM OVERLAPPING PLATES

m	P.E.		m	P.E.	
	log $N(m)$	Mag.		log $N(m)$	Mag.
8	±0.08	±0.24	12	±0.04	±0.12
9	.07	.21	13	.04	.12
10	.04	.12	14	.03	.09
11	±0.04	±0.12	15	±0.03	±0.09

ency of the magnitude system is assured. Except possibly for the very brightest stars, the magnitudes and the resulting counts may be considered to be in error by no more than 0.2 mag.

COMPARISON WITH "GRONINGEN PUBLICATIONS," NO. 43

Regions of nearly the same area as those used by van Rhijn in forming his tables of stellar surface distribution are compared with the present counts in Table 3. Differences log N(McCuskey)

TABLE 3

COMPARISON OF THE PRESENT COUNTS WITH *Groningen Publications*, NO. 43, TABLE 1
log N(McC.) *minus* log N(*G.P.*, No. 43)

m	SELECTED AREA					MEAN
	47	48	49	22	24	
8	−0.03	−0.03	0.00	+0.16	+0.03	+0.03
9	+ .16	+ .16	− .08	+ .10	+ .02	+ .07
10	+ .06	+ .17	− .04	− .03	− .07	+ .02
11	+ .08	+ .12	− .03	− .01	+ .10	+ .05
12	− .01	+ .01	+ .01	.00	+ .27	+ .05
13	.00	− .01	+ .07	+ .02	+ .01	+ .02
14	− .02	+ .02	− .01	− .05	− .02	− .02
15	+0.05	+0.03	+0.01	+0.02	−0.03	+0.02

minus log N(van Rhijn) have been formed for the magnitude range 8–15 in five Selected Areas. Table 1 of *Groningen Publications*, No. 43, which gives the actual unsmoothed values of log $N(m)$, has been used.

The agreement between the present counts and those of van Rhijn for the same areas is quite satisfactory. In Selected Areas 47 and 48 there is obviously a tendency for the RL counts of bright stars to exceed those of van Rhijn—an effect which might be attributed to the possible zero-point uncertainty in the present magnitudes were it not for the fact that the remaining areas do not show it. No systematic magnitude differences between these areas could be found to explain the tendency. The single large residual, +0.27, in Selected Area 24 arises because of an abrupt discontinuity in van Rhijn's data at this point.

THE SURFACE DISTRIBUTION OF STARS IN APPARENTLY UNOBSCURED REGIONS

A necessary starting-point in the analysis of star counts for the distance and absorbing power of nebulous material is a knowledge of the space-density function for unobscured regions at the same galactic latitude as the nebula and preferably near it in galactic longitude. To provide such comparison areas, regions of 3.1 square degrees each, in zones 2.5° apart in latitude, were chosen at longitudes where the stellar distribution appeared to be uniform and free of obscuring matter. The thirty-eight regions, ranging in latitude from 0° to −30°, were picked by inspection of the RL plates used for the star counts. As large a range in longitude as the counted area permitted was included, and several instances of nearly coincident areas were used as checks on the values of log $N(m)$. The mean longitude of the areas is 134°.7.

The resulting surface distribution of stars is given in Table 4. Figures 1 and 3 indicate the approximate positions of the apparently unobscured areas in this region of the sky and their relation to the total counted area. Mean values of log $N(m)$ are given for each zone. Galactic coordinates are referred to the Harvard pole, α 12^h40^m, δ +28°.

An inspection of Table 4 will indicate the general conformity of

TABLE 4

SURFACE DISTRIBUTION OF STARS IN APPARENTLY UNOBSCURED AREAS IN TAURUS

log $N(m)$ per Square Degree

AREA	λ	β	m							
			8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0
	Zone 0°; Area 9.3 Square Degrees									
1.....	149°.0	− 0°.1	0.19	0.66	1.13	1.51	1.83	2.14	2.43	2.72
43.....	150.5	− .4	0.30	.80	1.23	1.57	1.88	2.20	2.51	2.84
4.....	143.0	+ 0.1	9.90	0.47	1.00	1.48	1.82	2.15	2.43	2.72
Mean.	147°.5	− 0°.1	0.13	0.64	1.12	1.52	1.84	2.16	2.46	2.76
	Zone −2°.5; Area 15.5 Square Degrees									
2.....	151°.5	− 2°.7	9.99	0.49	0.97	1.40	1.78	2.08	2.34	2.61
42.....	151.6	2.6	0.20	.61	1.04	1.41	1.74	2.05	2.34	2.60
3.....	142.6	2.0	9.49	.17	0.80	1.38	1.80	2.15	2.48	2.80
6.....	141.8	2.0	0.13	.58	1.01	1.44	1.81	2.15	2.48	2.80
5.....	125.8	− 2.1	9.53	0.22	0.85	1.33	1.69	2.00	2.34	2.68
Mean.	140°.5	− 2°.2	9.86	0.40	0.92	1.39	1.76	2.09	2.41	2.72
	Zone −5°; Area 9.3 Square Degrees									
8.....	142°.1	− 4°.5	0.33	0.70	0.95	1.20	1.58	1.93	2.30	2.67
41.....	152.4	5.2	.26	.62	1.01	1.38	1.75	2.12	2.50	2.87
35.....	120.3	− 3.7	0.30	0.62	0.98	1.31	1.67	2.00	2.35	2.70
Mean.	138°.3	− 4°.5	0.30	0.65	0.98	1.30	1.67	2.02	2.38	2.75
	Zone −10°; Area 5.9 Square Degrees									
7.....	148°.4	− 9°.6	9.95	0.35	0.75	1.17	1.54	1.89	2.21	2.56
10.....	135.6	9.9	0.03	.40	.79	1.14	1.50	1.87	2.23	2.60
38*.....	109.5	−10.5	0.09	0.44	0.80	1.20	1.60	2.00	2.37	2.75
Mean.	142°.0	− 9°.7	9.99	0.37	0.77	1.16	1.52	1.88	2.22	2.58

* These regions were not included in the mean for the zone because of the wide disparity in galactic longitude.

TABLE 4—*Continued*

Area	λ	β	m							
			8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0
	Zone −12°.5; Area 9.3 Square Degrees									
9.....	147°.8	−12°.0	9.76	0.20	0.64	1.10	1.48	1.81	2.16	2.49
11.....	135.1	12.4	9.81	.24	.65	1.06	1.45	1.81	2.19	2.54
12.....	133.7	12.4	9.78	.36	.80	1.18	1.51	1.88	2.22	2.59
37*.....	110.0	−13.0	9.67	0.28	0.78	1.20	1.60	1.97	2.36	2.75
Mean.	138°.9	−12°.2	9.78	0.27	0.70	1.11	1.48	1.83	2.19	2.54
	Zone −15°; Area 3.1 Square Degrees									
14.....	131°.1	−14°.4	0.15	0.47	0.80	1.10	1.44	1.78	2.10	2.50
36*.....	110.0	−15.7	0.20	0.51	0.80	1.10	1.44	2.07	2.37	2.65
Mean†	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	Zone −17°.5; Area 9.3 Square Degrees									
13.....	140°.0	−18°.0	9.92	0.38	0.80	1.20	1.50	1.78	2.01	2.42
15.....	134.9	17.2	9.75	.21	.68	1.11	1.48	1.73	2.01	2.46
26.....	123.3	−18.0	9.51	0.20	0.79	1.20	1.51	1.69	2.09	2.48
Mean.	132°.7	−17°.7	9.73	0.26	0.76	1.17	1.50	1.73	2.04	2.45
	Zone −20°; Area 12.4 Square Degrees									
24.....	121°.8	−19°.8	9.94	0.30	0.65	1.00	1.37	1.67	2.00	2.46
22.....	130.3	20.0	0.25	.53	.81	1.13	1.44	1.78	2.12	2.50
17.....	133.0	19.7	9.80	.31	.79	1.17	1.51	1.76	2.07	2.44
16.....	138.8	−20.0	9.90	0.30	0.68	1.05	1.35	1.60	1.89	2.30
Mean	131°.0	−19°.9	9.97	0.36	0.73	1.09	1.42	1.70	2.02	2.42
	Zone −22°.5; Area 12.4 Square Degrees									
18.....	138°.2	−22°.3	9.89	0.34	0.75	1.12	1.45	1.70	2.00	2.35
19.....	130.0	22.2	0.09	.40	.72	1.06	1.35	1.61	1.92	2.29
23.....	130.8	22.0	0.09	.38	.69	1.03	1.41	1.80	2.16	2.50
25.....	121.7	−22.5	9.62	0.29	0.71	1.00	1.29	1.57	1.90	2.42
Mean.	130°.3	−22°.3	9.92	0.34	0.72	1.05	1.37	1.67	1.99	2.39

† Area 14 used individually.

TABLE 4—*Continued*

Area	λ	β	m							
			8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0
	Zone −25°; Area 9.3 Square Degrees									
20.....	135°.3	−24°.8	0.28	0.50	0.80	1.15	1.48	1.63	1.80	2.20
21.....	130.0	24.9	9.55	.20	.74	1.10	1.35	1.58	1.80	2.28
28.....	120.1	−25.5	0.15	0.44	0.71	1.00	1.17	1.38	1.80	2.32
Mean.	128°.5	−25°.1	9.99	0.38	0.75	1.08	1.33	1.53	1.80	2.27
	Zone −27°.5; Area 6.2 Square Degrees									
29.....	126°.0	−27°.5	9.80	0.40	0.80	1.09	1.33	1.59	1.84	2.33
30.....	131.0	−27.4	9.60	0.20	0.75	1.09	1.29	1.49	1.70	2.25
Mean.	128°.5	−27°.5	9.70	0.30	0.78	1.09	1.31	1.54	1.77	2.29
	Zone −30°; Area 6.2 Square Degrees									
32.....	122°.0	−29°.1	9.70	0.10	0.50	0.88	1.22	1.57	1.80	2.29
31.....	129.5	−29.7	9.75	0.28	0.74	1.15	1.40	1.56	1.72	2.21
Mean.	125°.7	−29°.4	9.73	0.19	0.62	1.01	1.31	1.56	1.76	2.25

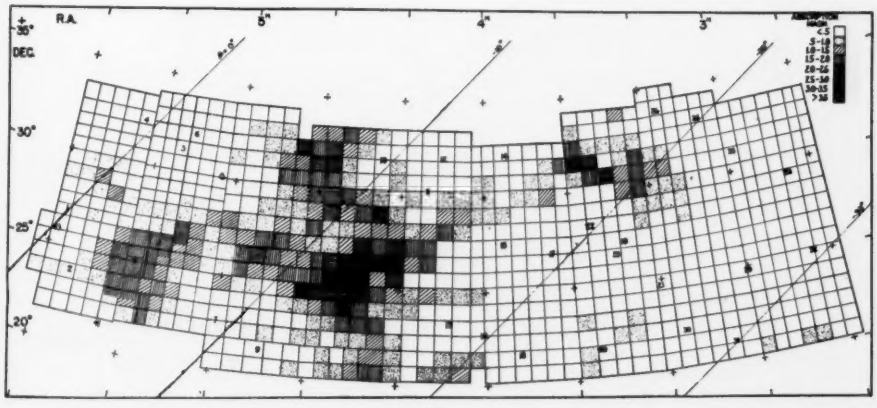


FIG. 1.—Distribution of obscuring matter in Taurus. The small numbers indicate the areas selected as unobscured. The absorption in magnitudes is indicated by the degree of shading.

the values of log $N(m)$ among the individual areas of each zone and their suitability as comparison regions for detailed studies of adjacent obscuring material.

A series of curves showing the variation of the log $N(m)$ values with galactic latitude is shown in Figure 2. Means from Table 4

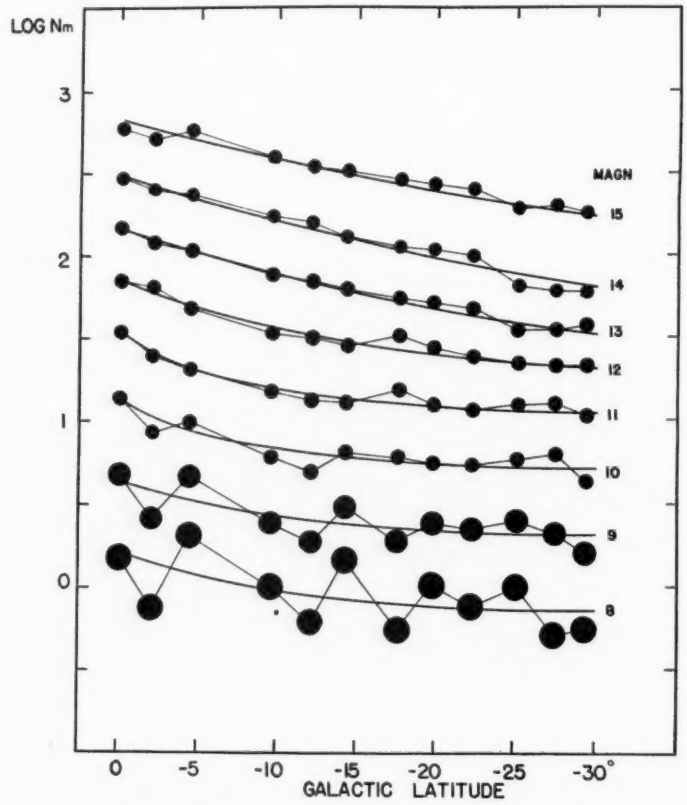


FIG. 2.—Variation of log $N(m)$ with galactic latitude. The probable errors in log N are indicated by the radii of the filled circles.

have been plotted, the size of the probable errors being indicated by the radii of the filled circles. No glaring inconsistencies are apparent from these data, and consequently the averaged values of log $N(m)$ will be used for a future discussion of the stellar space-density distribution and the density variation with distance from the galactic plane.

COMPARISON OF THE SURFACE STELLAR DISTRIBUTION WITH THAT OF EARLIER INVESTIGATORS

Before proceeding to an analysis of the counts for space density, it will be well to compare them with similar data on mean distributions by van Rhijn[7] and by Seares and Joyner,[8] and with the detailed data of von Klüber.[9] A test of the accuracy of the counts for the bright stars is furnished by the catalogue of photographic magnitudes published in the *Harvard Mimeograms*, Series I.

a) Comparison with *Groningen Publications*, No. 43, Table 10, may be obtained by considering Table 5, in which residual differences between the present data and van Rhijn's mean distribution tables are shown. Differences have been formed in the sense log N(McC.) *minus* log N(*G.P.*, No. 43) for zones of galactic latitude between 0° and −30° and for photographic magnitudes from 8 to 15. Appropriate corrections for differences in galactic coordinates have been made, and the data of van Rhijn's Table 10 have been taken for $\lambda = 135°$.

TABLE 5

COMPARISON OF LOG $N(m)$ WITH TABLE 10, *Groningen Publications*, NO. 43

log N(McC.) *minus* log N(*G.P.*, No. 43)

GAL. LAT.	m								MEAN
	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0	
0°...	+0.22	+0.20	+0.18	+0.24	+0.30	+0.25	+0.17	+0.10	+0.21
− 5....	.21	.20	.14	.18	.23	.23	+ .24	.20	.20
−10....	.19	.18	.16	.18	.26	.23	+ .24	.25	.21
−15....	.17	.19	.18	.18	.19	.19	+ .21	.31	.20
−20....	.17	.23	.22	.17	.15	.12	+ .18	.31	.19
−25....	.15	.23	.21	.17	.11	.05	+ .07	.26	.16
−30....	+0.15	+0.25	+0.26	+0.21	+0.10	+0.00	−0.03	+0.23	+0.15

Two facts are evident immediately from an inspection of Table 5: (1) there is a remarkably constant systematic difference, amounting in the mean to +0.19 in log $N(m)$, between the two sets of data; and (2) this systematic effect does not depend upon magnitude and diminishes only slightly with galactic latitude as far as $\beta = -30°$.

[7] *Groningen Pub.*, No. 43, Table 10, 1929.

[8] *Mt. W. Contr.*, No. 346; *Ap. J.*, **67**, 24, 1928.

[9] *Zs. f. Ap.*, **6**, 259, 1933.

The latter, of course, precludes scale differences in the magnitude systems used, while a zero-point discrepancy of the size necessary to account for the phenomenon would amount to 0.7 mag. Such a relative error in the magnitude systems is quite unlikely, and, indeed, it has been shown that the RL counts for the Selected Areas themselves agree, within the observational errors, with the actual counts made by van Rhijn.

One may infer, therefore, that a general veil of obscuring nebulosity absorbing about 0.7 mag. is present over the Selected Area regions, and consequently the distribution tables based upon these areas have yielded low values for $\log N(m)$. An inspection of the RL plates shows that all the Selected Areas are either in or on the edge of visible areas of nebulosity. None of them falls in the regions selected for the present analysis as unobscured. This dearth of stars is confirmed by a comparison of the calibration curves for each RL plate as a whole with the curves for the unobscured regions. Again a residual excess of nearly 0.2 in $\log N(m)$ is shown by the latter over the mean $\log N(m)$ for the whole plate.

In his study of the Cygnus region, Miller[10] has found a similar deficiency in $\log N(m)$ for the van Rhijn tables, but in this region the discrepancy became insignificant for the zone at $+15°$ in galactic latitude. In the case of the present counts, however, the flare of nebulous material extends to much higher latitudes. Even at $\beta = -20°$ to $-30°$ the diminution in the general absorption has not become very great. This large flare, evident also in Hubble's zone of avoidance of extragalactic objects,[11] and in the distribution of the bright galaxies of the Shapley-Ames catalogue,[12] is the striking characteristic of the Taurus region.

b) Seares and Joyner,[8] in 1928, published tables of deviations which were to be applied to the mean star counts for various zones of galactic latitude in order to obtain values of $\log N(m)$ for any given galactic longitude. If the resulting logarithms, with corrections suitable to galactic longitudes 120°–150° already applied, are compared with the data for unobscured areas (Table 4), it is found that the Seares tables also yield too low values of $\log N(m)$. In

[10] *Harvard Ann.*, **105**, 305, 1937.

[11] *Mt. W. Contr.*, No. 485; *Ap. J.*, **79**, 41, 1934.

[12] *Harvard Ann.*, **88**, No. 2, 1932.

Table 6, differences log N(McC.) *minus* log N(*Mt. W. Contr.* No. 346) are shown for latitudes from 0° to −30° and magnitudes 9–15.

An inspection of Table 6 reveals clearly two groups of residuals: (1) those for stars with $m < 13$, which average +0.39 in $\Delta \log N(m)$, and (2) those for stars with $m \geqq 13$, which average +0.15 in $\Delta \log N(m)$. The grouping is significant in that the boundary marks the limiting magnitude between the two sets of data used by Seares and Joyner in forming their tables, viz., for stars with $m < 13$ the *Astrographic Catalogues* were used; for stars with $m > 13$ the *Mount Wilson Catalogue of Selected Areas* was used. For stars fainter than magnitude 13 the data used by Seares and Joyner and by van Rhijn

TABLE 6

COMPARISON OF LOG $N(m)$ WITH THE DATA OF SEARES AND JOYNER

log N(McC.) *minus* log N(*Mt. W. Contr.*, No. 346)

Gal. Lat.	m						
	9.0	10.0	11.0	12.0	13.0	14.0	15.0
0°	+0.40	+0.49	+0.55	+0.48	+0.31	+0.19	+0.12
−10	.42	.43	.43	.38	.24	+ .15	.10
−20	.35	.36	.33	.26	.14	+ .14	.31
−30	.37	.41	.36	.28	.09	− .03	.13
Mean	+0.38	+0.42	+0.42	+0.35	+0.19	+0.11	+0.16

were derived from a common source, and consequently the deficiencies found in these data as compared with the present counts should agree on the average, as they do. In the case of the brighter stars the use of the *Astrographic Catalogue* material has apparently led to large systematic errors in the distribution tables of Seares and Joyner, as has been already pointed out by Lindsay and Bok.[13]

c) Von Klüber's[9] star-count analysis for a small portion of the Taurus region (33 square degrees at $\lambda = 137\overset{\circ}{.}7$, $\beta = -12\overset{\circ}{.}7$; limiting magnitude 14.5 photographic) has been based on three comparison regions (*a*, *b*, and *c*) selected by inspection of Barnard's *Atlas* as being free of obscuring clouds. In his analysis of these regions, however, von Klüber came to the conclusion that they were affected by obscuring material, the obscuration amounting to 0.8 mag. in the mean.

[13] *Harvard Ann.*, **105**, 287 ff., 1937.

A test of this result has been made by forming differences log N(McC.) *minus* log N(von Klüber) between the present counts for the latitude zone in which the comparison areas lie and von Klüber's values for these areas. The residuals are shown in Table 7.

Not only is the mean residual of Table 7 high— +0.34 in log $N(m)$ —but there is a serious scale difference which may be attributed to possible scale errors in the magnitudes used by von Klüber. The absorption given by the mean residual for the three regions is approximately one magnitude, which agrees fairly well with the absorption found by von Klüber.

TABLE 7

COMPARISON OF LOG $N(m)$ WITH VON KLÜBER'S DATA

log N(McC.) *minus* log N(von K.)

REGION	m								MEAN
	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0	
a.....	+0.16	+0.30	+0.28	+0.20	+0.09	+0.00	+0.06	+0.09	+0.15
b.....	.64	.62	.55	.45	.39	.26	.18	.16	.41
c.....	+0.90	+0.83	+0.68	+0.53	+0.30	+0.10	+0.05	+0.10	+0.44

d) The large systematic excess of the RL counts for the brightest stars over those of the previous investigators has indicated the necessity for one further test of the brighter end of the magnitude sequence used in the present investigation. Dr. C. H. Payne-Gaposchkin has published accurately determined photographic magnitudes for stars brighter than $m = 8.25$ in *Harvard Mimeograms*, Series I, No. 2; a portion of the area so covered is included in the Taurus area counts.

In this catalogue, counts ($m \leqq 8.25$) were made for areas of 32 square degrees each, ranging in latitude from $+5°$ to $-30°$. The resulting values of log $N(m)$ were compared with the values taken from the smoothed curves of Figure 2, and the residuals are exhibited in Table 8.

It is apparent from an inspection of these residuals that the scale and zero point of the magnitudes adopted for the RL counts differ very little from those of the *Harvard Mimeograms*. Since the latter is presumably on the International Scale, we may consider the

present counts to conform closely to this standard. The serious departures noted in comparisons (*b*) and (*c*) above are in all probability not due to errors in the present magnitude system.

GENERAL ABSORPTION IN THE TAURUS REGION

An inspection of curves relating $\log A(m)$, where $A(m)$ is the number of stars per square degree between $m - \frac{1}{2}$ and $m + \frac{1}{2}$, and m, for several areas imbedded in the Taurus nebulosity, shows that in nearly every case the absorption has become complete before stars of the twelfth magnitude are reached. In view of this circumstance, a picture of the distribution of absorbing matter over the entire counted area may be obtained by studying the deficiencies of fifteenth-magnitude stars in small subareas as compared with the appropriate unobscured regions of Table 4.

TABLE 8

COMPARISON OF BRIGHT STAR COUNTS WITH *Harvard Mimeograms*, SERIES I, NO. 2

log N(McC.) *minus* log N(*H. Mimeo.*)

Average β	$\Delta \log N(m)$	Average β	$\Delta \log N(m)$
− 1°.0	−0.05	−17°.5	−0.07
− 6.5	+ .12	−27.2	− .02
−12.2	+0.08	−31.5	+0.06

For this purpose the original values of $\log N(m)$ for $m = 15$ were arranged in subareas of 0.69 square degrees each and compared with the corresponding values of $\log N(m)$ for the regions chosen as unobscured. Differences log N(unobscured) *minus* log N(subarea) for each 2°.5 zone of galactic latitude were plotted on a chart of the region. Mean values of $\log N(15)$ from Table 4 were used directly for the reference background. If the deficiencies in $\log N(m)$ so found for the subareas are attributed to absorbing nebulosity, the amount of the latter may be expressed in magnitudes by multiplying the $\Delta \log N(m)$ by three. This conversion factor was obtained from the m, $\log N(m)$ curves for nineteen plates and was found to be independent of galactic latitude.

Figure 1 presents the resulting distribution of obscuring material over the Taurus region. The squares indicated on the map represent

approximately the subareas into which the region was divided for analysis. The absorption in intervals of 0.5 mag. is indicated by varied degrees of shading.

The most conspicuous features of the Taurus region are the three great concentrations of dark nebulosity at 3^h40^m, $+32°$; 4^h30^m, $+26°$; and 5^h20^m, $+25°$. These well-known dark areas were first outlined in detail by Dyson and Melotte[14] in 1919. Figure 1 agrees well with the map of Taurus published by these authors.

At 4^h30^m, $+26°$, the absorption reaches a maximum of 3.7 mag., a value which compares favorably with that found by Dyson and Melotte, 3.9 mag. Von Klüber[9] has estimated that the absorption reaches 4 mag. in the darkest part of the region.

For the other two intensely obscured areas—5^h20^m, $+25°$, and 3^h40^m, $+32°$—the absorption found in the present analysis averages 2.4 mag., as compared with the value 2.2 mag. deduced from the data of Dyson and Melotte, and the value 2.5 mag. given by von Klüber.

It is evident from Figure 1 that the areas of small obscuration are quite extended—a fact which may be seen more clearly by a study of Figure 3, which shows the same data as Figure 1 in the form of a contour map. Contours have been drawn on the map at intervals of 0.2 mag. absorption, the integral values being indicated by heavy lines. The concentrations of high obscuration mentioned above are conspicuously shown by the crowded contours. Figure 3 makes a rapid determination of the total absorption of photographic light for any small area a matter of inspection. Data of this kind may be useful in computations of distance moduli which depend upon photometric measures of variable stars, etc.

Both Figures 1 and 3 display the regions selected by inspection of the RL plates as being unobscured. In only one case, that of Area 2, is there definite evidence of appreciable absorption over one of these regions. Areas 5, 35, 36, 37, and 38 of Table 4 have not been shown in Figures 1 and 3. They are situated near the galactic equator at δ $+45°$.

The evidence of an excess of stars in these so-called "unobscured regions" as compared with the general data of others, together with

[14] *M.N.*, **80**, 3, 1919.

the smallness of the residuals shown by Figures 1 and 3 for subareas of the regions, points to their suitability as comparison areas for determining the distances and absorbing powers of the interspersed nebulae. Furthermore, the space densities computed from the mean values of log $N(m)$ of Table 4 may be taken as representative of the general galactic structure in this anti-center region of the galaxy.

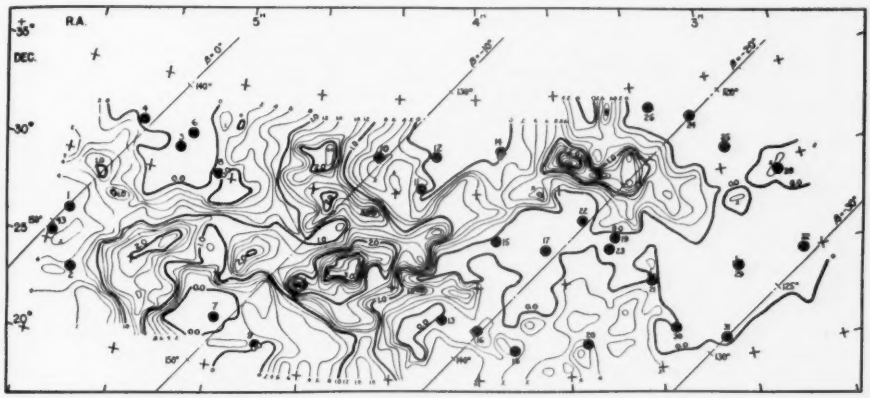


FIG. 3.—Detailed distribution of nebulosity in Taurus. Black circles are areas selected as unobscured. The contour interval is 0.2 mag.

Since the present counts extend over a large range in galactic latitude, the density variation perpendicular to the galactic plane may be deduced. It is planned in succeeding communications to present these space-density analyses for the unobscured as well as for the obscured regions and to investigate by means of color-spectral class relations certain of the more interesting parts of the Taurus nebula.

In conclusion, I wish to express my sincere appreciation for the hearty co-operation of Dr. Harlow Shapley, Dr. Bart J. Bok, and Mr. Louis A. Brigham in furnishing the observational material upon which the Taurus investigation is based. To Dr. Bok I also wish to express my indebtedness for many helpful conferences relative to the problems of galactic structure.

WARNER AND SWASEY OBSERVATORY
CASE SCHOOL OF APPLIED SCIENCE
June 15, 1938

THE PRESENT SPECTRAL CHARACTERISTICS OF SIXTEEN OLD NOVAE*

M. L. HUMASON

ABSTRACT

Spectra of sixteen old novae have been observed with a small-dispersion spectrograph. Observations have been made in the fields of two others in an effort to identify them by their spectra. Visual magnitudes of the observed objects range from 10.0 to 16.5. Charts of all the fields are given for the benefit of other observers.

Most of the objects have a strong continuous spectrum extending well into the violet, in which neither absorption nor emission lines can be seen. They have therefore been classified as O Con, since the extension into the violet corresponds to that of O-type stars, or possibly of early B stars. When emission lines have been observed, the stars have been classified as O Em.

In the search for Nova B Cassiopeiae (1572), Baade observed an interesting faint blue star on a color plate of the region. Its photographic magnitude is 17.0, its spectrum Aon. As the mean absolute magnitude for stars of this type is roughly +1.0, the distance modulus, uncorrected for space absorption, is 16.0, thus indicating a distance of the order of 16,000 parsecs in a direction 125° from that of the galactic center. It is not assumed that this star is the nova.

Spectral types of four stars observed in the field of Nova Vulpeculae (1670) indicate that this nova has not yet been rediscovered. One of these objects is the faint variable star found by W. H. Steavenson.

With the exception of T Coronae, all the novae observed are blue and probably correspond in temperature with the O or early B stars. T Coronae may also be blue if it is the companion of a red star.

The density of Nova Persei (1901) in its present state is of the order of 220 ⊙. The corresponding value for Nova Aquilae (1918) is 70 ⊙. For eleven novae whose distances are not so reliably known, a mean density has been determined by using $M_{max} = -7.0$, the mean M of galactic novae at maximum, from which their present mean M is obtained by adding the mean of their amplitudes. The resulting mean density of this group is 60 ⊙.

During the past several years spectrograms of sixteen old novae have been obtained. In addition, stars in the fields of two other novae have been observed, in an effort to identify them by their spectra. The instrument used is the 100-inch Cassegrain nebular spectrograph, which has a dispersion of 500 A/mm at $H\gamma$. Visual magnitudes are used throughout this paper unless otherwise stated.

The mean magnitude of the novae observed is about 14.0. Whenever the object was bright enough, its spectrum was widened by allowing the star to drift along the slit. While most of the spectra are narrow and the dispersion small, the observations give for the first

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington,* No. 596.

time an indication of the present spectral characteristics of these objects.

Of the eighteen objects observed, fifteen are thought to be correctly identified.[1] Two others, Nova B Cassiopeiae (1572) and Nova Vulpeculae (1670), are probably not yet known; and a third, Nova Sagittarii (1898), is still doubtful. Charts of all objects observed are reproduced from direct photographs kindly obtained for the writer by Messrs. Baade and van Maanen with either the 60- or 100-inch

TABLE 1

NOVAE OBSERVED

Nova	α 1900	δ 1900	m_v Max.	m_v Min.	Sp.
B Cass (1572)	0^h19^m2	+63°36′	−5?	?	?
Per (1901)	3 24.4	+43 34	0.0	11.8–14.0	O Em
T Aur (1891)	5 25.6	+30 22	4.5	14.8	O Em
Gem (1903)	6 37.8	+30 3	7.3	16.5	O Con
Gem (1912)	6 48.4	+32 16	3.7	14.7	O Em
T Pyx (1890–1902–1918)	9 0.5	−31 59	7	14.1	O Em
T Cor B (1866)	15 55.3	+26 12	2.0	10.6	? Em
Oph (1848)	16 53.9	−12 44	5.0	12.2–13.0	O Con
RS Oph (1898–1933)	17 44.8	− 6 40	4.3	11.8	O Em
Sgr (1919)	18 25.7	−29 28	7	14.0	O Con
Aql (1918)	18 43.8	+ 0 28	−1.4	10.8	O Em
Lyr (1919)	18 49.5	+29 6	6.5	15.2	O Con
Sgr (1898)	18 56.2	−13 8	4.7	16.5	O Con
Vul (1670)	19 43.5	+27 4	3	?	?
Cyg (1920)	19 55.9	+53 21	2.0	15.5	O Em
Sge (1913)	20 3.1	+17 24	7.2	15.2	O Con
Q Cyg (1876)	21 37.8	+42 23	3	14.4–15.2	O Em
Lac (1910)	22 31.8	+52 12	5.0	14.4	O Con

reflector (Pls. VII–X). All the photographs have been enlarged to approximately the same scale, about 7″.5 per millimeter. Since the exposures varied from 10 to 60 minutes, the images of the different objects do not indicate their relative brightness.

Table 1 lists the novae observed, their positions and visual magnitudes,[2] and their spectral types, which are approximate because of the small dispersion used. All have been assigned to two groups designated as O Em, or type O with emission lines present; and O Con,

[1] The writer would greatly appreciate being informed of any known misidentifications.

[2] Most of the magnitudes in the fifth column are measures by W. H. Steavenson which appear in his yearly reports of "Observations of Novae" in *Monthly Notices*.

or type O without observable absorption or emission in what appears to be a simple continuous spectrum. The relative intensity of a continuous spectrum extending well into the blue region, and the absence of absorption lines, especially of calcium H and K, were the criteria for classification as type O.

NOVA B CASSIOPEIAE (1572)

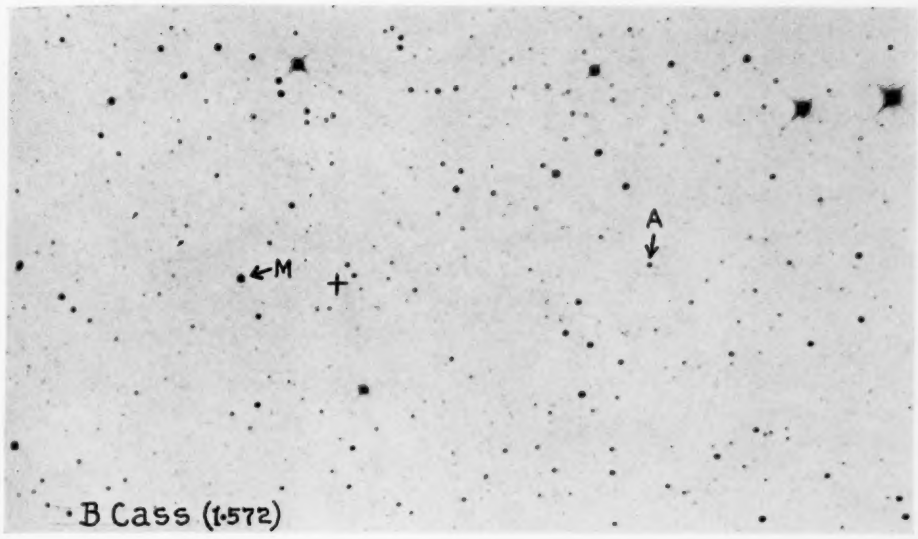
Two stars have been observed in the field of this nova: one by Lundmark and the writer in 1922, and one by Baade and the writer in 1937. The 1922 observations were made with a slitless spectrograph attached to the Newtonian focus of the 60-inch reflector, and covered a field 40′ square. Spectra of fourteenth-magnitude stars were registered on the longest exposure, but none could definitely be identified as that of the nova. The spectrum of an M-type star appeared very close to the computed position of the nova, however, and was noted for the reason that at that time (1922) the spectrum of another nova, T Coronae, was believed to be of type Mb pec. There are now sound reasons for believing that T Coronae is not an M-type star, however; and[3] it is therefore extremely improbable that the M-type star mentioned is B Cassiopeiae.

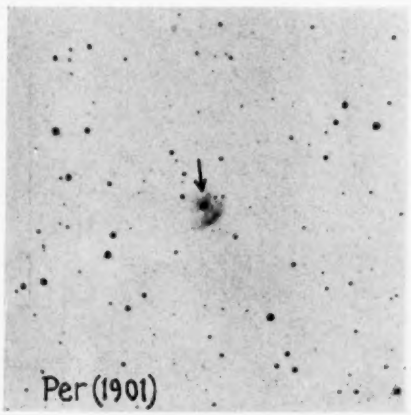
More recently Baade and the writer have again attempted to identify this nova. Assuming that B Cassiopeiae was a supernova with a large range in brightness, and that its present spectrum should be that of an O- or early B-type star, Baade is searching for faint blue stars[4] near the computed position of the nova. So far, one color plate has been obtained, on which the limiting photographic magnitude at the center of the field is about 18. In an area 21′ in diameter, only one blue star has been found. Baade estimates its photographic magnitude as 17.0 and its color index as 0.0. Its position for 1865 is: right ascension, 0^h16^m4; declination, $+63°23'$. A very good spectrogram of this star obtained by the writer in November, 1937, indicates that it is probably not B Cassiopeiae but is interesting, nevertheless, in its own right on account of its type and the resulting probable distance. The spectrum has been classified as Aon, as the hydrogen lines are strong and fairly wide and the calcium lines H

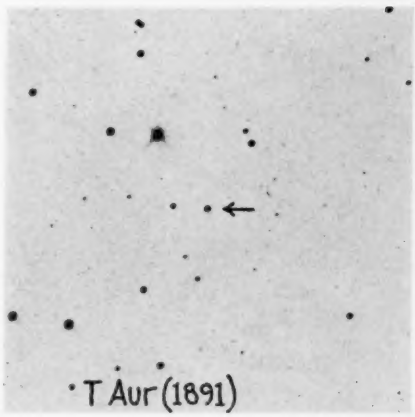
[3] See description of the spectrum of T Coronae on p. 233.

[4] *Mt. W. Contr.*, No. 600; *Ap. J.*, **88**, 285, 1938.

PLATE VII







FIELDS OF NOVAE
Scale, 1 mm = 7″.5

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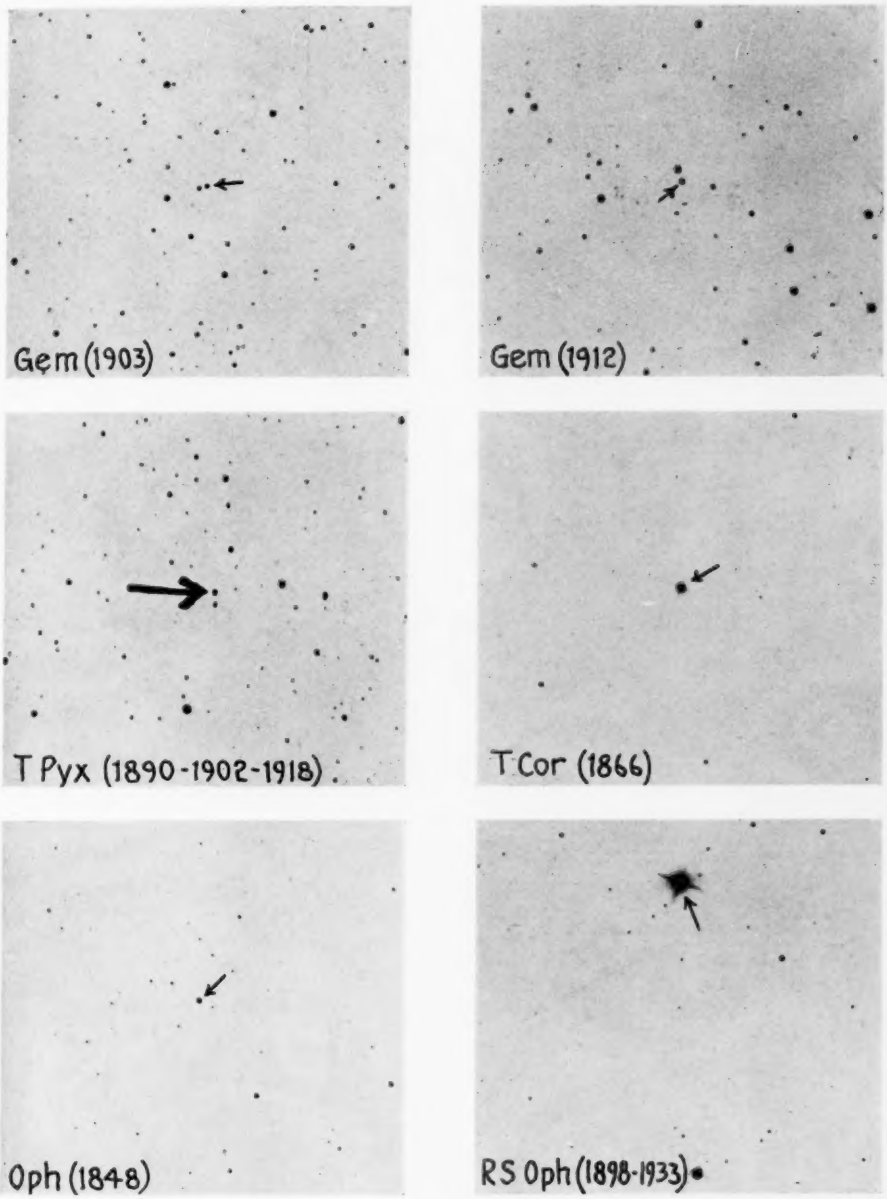
PLATE VIII



FIELDS OF NOVAE
Scale, 1 mm = 7″.5

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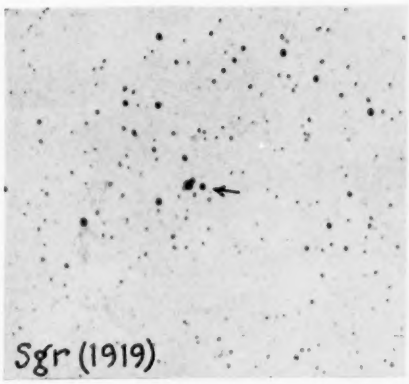
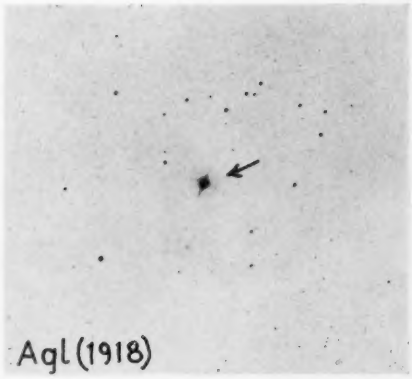
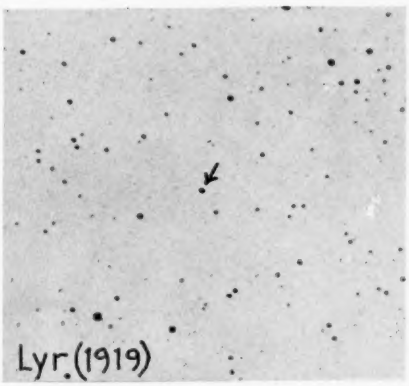
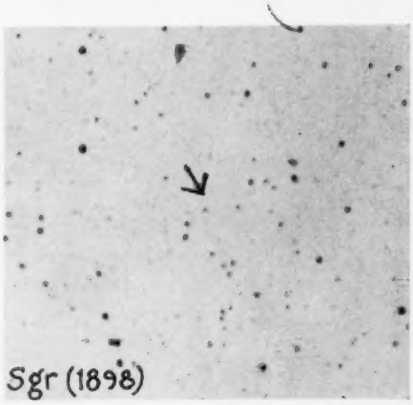
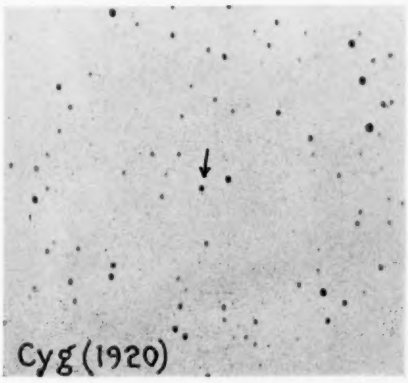
PLATE IX

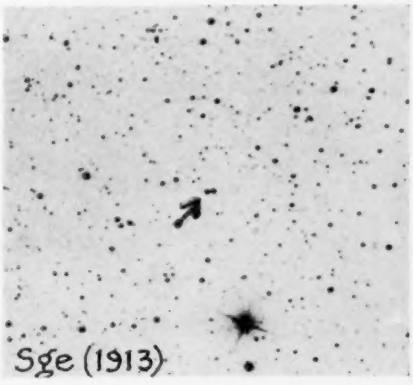












FIELDS OF NOVAE

Scale, 1 mm = 7″.5

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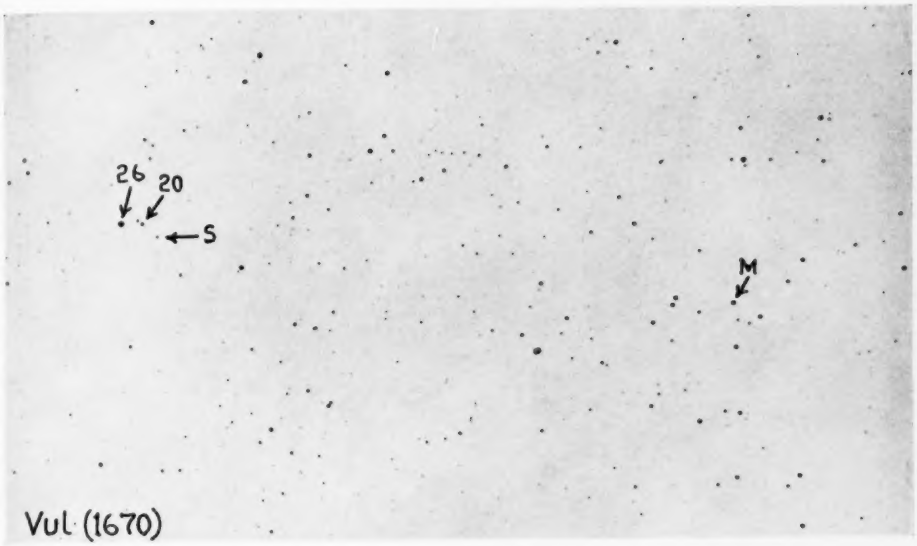
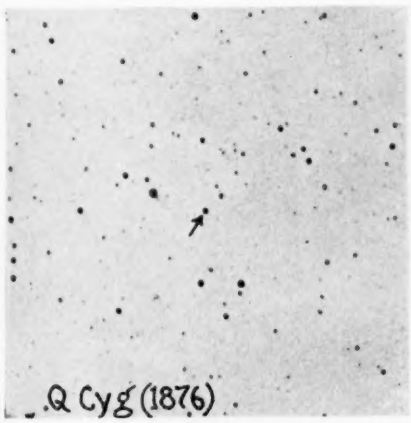
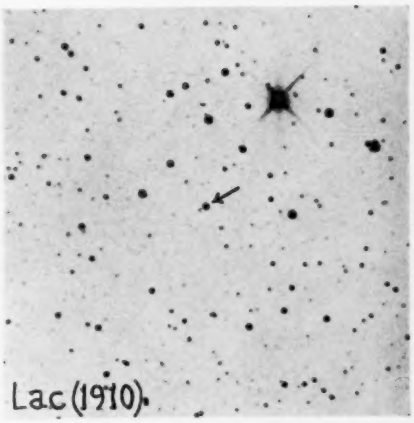
PLATE X







FIELDS OF NOVAE
Scale, 1 mm = 7″.5

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and K do not show. The strength of the hydrogen lines and the fact that they are not unduly wide indicate that the star is not a white dwarf but a normal A-type giant. The mean absolute magnitude of stars of this type is roughly +1.0, which gives for the distance modulus, uncorrected for space absorption, the value 16.0, thus indicating a distance of 16,000 parsecs in a direction 125° from that of the galactic center.

On the chart of the field of B Cassiopeiae (Pl. VII) the M-type star observed in 1922 is designated by "M," and the faint A star found by Baade by "A." The cross marks the computed position of the nova for 1922. The search for this nova will be continued over a somewhat larger area and to a fainter limiting magnitude in the near future.

NOVA PERSEI (1901)

This is one of the most interesting novae observed. Thirty-seven years after the outburst, its light is still variable over a range of 2 mag., and minor changes in the intensities of the emission lines also occur. It is also one of two novae having an expanding shell of gas which is bright enough to be photographed, the other being Nova Aquilae (1918). The shell now has a radius of 20″; and since 1917, when the nebulosity was first discovered, it has expanded at the uniform rate of 0″.4 per year.[5] The spectra of both the nova and the shell contain emission lines; but the only element common to both is hydrogen, which is strong in the nova and extremely weak in the spectrum of the gaseous envelope. Table 2 lists the lines observed in the nebula and in the nova, with their mean intensities as observed on eight Imperial Eclipse plates.

Emission lines in the spectrum of the nova are of the order of 16 A wide and should be classified as bands. Their intensities for the nova spectrum observed on the individual spectrograms are given in Table 3. Except on the 1933 spectrograms, λ 4686 appears to be the strongest emission feature in the spectrum. The lines of ordinary helium are weak and disappear altogether at times. $H\gamma$ is usually the strongest hydrogen line, although sometimes $H\delta$ becomes almost equal to, and on January 30, 1930, was stronger than, $H\gamma$. There seems to be no outstanding correlation between the variation in the

[5] *Pub. A.S.P.*, **46**, 229, 1934.

intensity of the emission lines and the variation in the light of the nova. The first six magnitudes in the table are Steavenson's values for the dates, or within a few days of the dates, on which the spectrograms were obtained. The last two are visual estimates made by the writer at the Cassegrain focus of the 100-inch reflector.

TABLE 2

EMISSION LINES OBSERVED IN THE SPECTRA OF THE NEBULA AND THE NOVA

NEBULA			NOVA PERSEI (1901)		
λ	Element	Int.	λ	Element	Int.
3727	*O* II	40	3835	*H*	2
3869	*Ne* III?	65	3889	*H*	3
3968	*Ne* III?	50	3970	*H*	3
4102	*H*	2	4026	*He* I	2
4340	*H*	5	4102	*H*	5
4363	[*O* III]	10	4340	*H*	6
4571	.....	1	4471	*He* I	1
4722	*Ne* IV?	2	4686	*He* II	6
4861	*H*	3	4861	*H*	4
4959	[*O* III]	15			
5007	[*O* III]	25			

TABLE 3

EMISSION-LINE INTENSITIES IN THE SPECTRUM OF NOVA PERSEI (1901)

Date	m_v	3835	3889	3970	4026	4102	4340	4471	4686	4861
1930, Jan. 30	12.3	.....	.....	2	1	7	5	0	7	5
1933, Oct. 17	13.8	.....	.....	2	0	2	3	0	1	1
1933, Nov. 18	13.5	2	5	5	3	6	8	1	3	2
1934, Dec. 2	13.1	1	3	5	3	8	10	2	8	6
1935, Nov. 25	12.9	1	2	3	1	3	6	2	8	2
1937, Jan. 13	12.7	1	3	4	2	3	5	0	5	2
1937, Nov. 4	13.2	1	2	3	2	7	7	2	8	3
1937, Nov. 5	13.2	1	2	3	2	5	7	2	8	5

NOVA T AURIGAE (1891)

According to Steavenson, the light of this nova has remained constant at magnitude 14.8. Spectrograms were obtained on November 19, 1933; January 13, 1937; and March 6, 1937. On all three dates the spectrum appears the same. A strong continuous spectrum extends well into the violet, with λ 4686 and *Hδ* showing as extremely weak emission lines, λ 4686 being slightly the stronger.

NOVA GEMINORUM (1903)

Nova Geminorum (1903) is one of the faintest novae observed. From its appearance on the slit of the spectrograph, the magnitude was estimated as 16.5. A single spectrogram obtained on October 21, 1933, shows an apparently continuous spectrum without observable absorption or emission lines. The extension into the violet is less pronounced for this nova than for the others.

NOVA GEMINORUM (1912)

The magnitude of this nova is now 14.7. A single plate taken on November 14, 1933, shows a fairly strong continuous spectrum, with $H\beta$, $H\gamma$, and $H\delta$ as weak emission lines, their intensities being only slightly greater than that of the continuous spectrum.

NOVA T PYXIDIS (1890–1902–1918)

This nova and RS Ophiuchi are the only novae known to have had more than one outburst. T Pyxidis on three occasions has brightened from magnitude $14\pm$ to a maximum magnitude of $7\pm$. On each occasion the light-curve and the spectrum were typical of other novae; but until the causes of the outbursts are better understood, both T Pyxidis and RS Ophiuchi should probably not be classified as typical novae.

The only spectrogram of T Pyxidis on which the exposure was sufficient was obtained on March 12, 1934, sixteen years after the last outburst. On this plate the continuous spectrum is fairly strong, and the following lines are bright (intensities in parentheses): λ 5007 (1), $H\beta$ (3), λ 4686 (10), $H\gamma$ (2), and $H\delta$ (2). On account of the star's southern declination, it is impossible to judge as to the extension of the continuous spectrum into the violet. Two extremely weak plates with higher dispersion were obtained on February 20, 1927, and March 16, 1929. On the latter plate λ 4686 can barely be seen and is the only emission feature visible. Only a faint trace of the continuous spectrum is visible on this plate.

NOVA T CORONAE (1866)

The spectrum of this nova has previously been described by Adams and Joy in 1921[6] and by Lundmark in the same year.[7] A spectrogram obtained on March 6, 1930, shows strong titanium

[6] Adams and Joy, *ibid.*, **33**, 263, 1921. [7] Knut Lundmark, *ibid.*, p. 271.

oxide bands and $H\beta$, $H\gamma$, and λ 4686 as emission lines. On June 6 and 7, 1938, two spectrograms were obtained by Joy which show the emission lines stronger and considerably narrower than previously observed.

For many years the spectrum has been considered to be of type Mb. In 1921, Adams and Joy[6] describe it as having

> strong titanium oxide bands. The hydrogen lines $H\beta$ and $H\gamma$ are bright and very broad, the latter probably showing a dark reversal, but with the violet component much the stronger. $H\delta$ is an absorption line. The most interesting feature is the presence of the bright helium line at λ 4686 so characteristic of Wolf-Rayet stars. It resembles $H\beta$ in appearance.

In addition, they state that the radial velocity of the star, as derived from the absorption lines, is -5 km/sec, while the bright lines, $H\beta$, $H\gamma$, and λ 4686, are displaced toward the violet, relative to the absorption lines, by about 1.3 A.

At that time very little was known about the excitation potential necessary to excite the line λ 4686, or about the later history of novae in general. Now, however, there are strong arguments against accepting the hypothesis that the M-type absorption spectrum is that of the nova. These, in their order of importance, are:

1. The temperature of an M-type star is far too low to excite a line with an excitation potential as high as that of λ 4686.

2. The bright lines have not the same displacement as the absorption lines.

3. The light-curve during the rise to maximum, and for a relatively short interval after maximum, is like that of other novae; but on the steep down-branch of the curve, at magnitude 10.0, the light of the star suddenly became constant and except for infrequent brightening of about half a magnitude, has since remained so.

4. Without exception, other novae which have been observed are now blue stars.

Consideration of these facts leads to the conclusion that the nova T Coronae is probably a faint blue companion of the M-type star, similar to the known faint blue companions of *o* Ceti, R Aquarii, and Z Andromedae.[8]

[8] *Note added September 9, 1938:* Since the recently reported increase in brightness of T Coronae, additional spectrograms have been obtained by Joy and by Minkowski. They find a decided increase in the strength of the continuous spectrum when compared

Acceptance of this conclusion does away entirely with any forced interpretation of the observed spectrum or of the light-curve. The emission lines presumably belong to the nova itself, which thus accounts for the presence of λ 4686 and for the difference in displacement between the bright and the dark lines. The peculiarity in the light-curve is also explained, and the light-amplitude becomes greater and therefore more nearly normal than has heretofore been supposed.

If the foregoing explanation is true, then all the old novae formerly classified as red stars should now be eliminated from future records. Nova Aquilae No. 4 (1920) is now generally believed to be a long-period or an irregular variable star of type R, not a nova. Reasons for the rejection of the other two, B Cassiopeiae and T Coronae, seem sound and are presented in this paper.

NOVA OPHIUCHI (1848)

Recent observations by Steavenson of the brightness of this nova show it to be variable over a range of 0.8 mag. from 12.2 to 13.0. A single spectrogram obtained June 18, 1936, shows a strong continuous spectrum extending well into the violet, with neither absorption nor emission lines discernible.

NOVA RS OPHIUCHI (1898–1933)

Of the two outbursts of this nova, the first maximum, given as 7.7, was not well determined and may not be the true maximum. During the second outburst the star reached a magnitude of 4.3. At minimum the light remains fairly constant around magnitude 11.8. Four spectrograms have been obtained, one before, two at the time of, and one three years after, the 1933 outburst. Only the first and last are described here, since spectra of higher dispersion obtained during the 1933 maximum have already been described in detail by other observers. One interesting feature concerning the spectrum during the 1933 maximum should be mentioned, however; namely, the discovery by Adams and Joy of five coronal lines in the

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with the 1921 spectrograms; and Minkowski's spectrograms, obtained with a Schmidt spectrograph, show that the spectrum to the violet of λ 3900± is essentially that of an early-type star. Both observers intend to publish detailed accounts of their observations in the near future.

spectrum of this star.[9] So far as known, this is the first instance in which these lines, previously observed only in the solar corona, have been observed in the spectrum of a nova.

The description of the first spectrogram, dispersion 175 A/mm at $H\gamma$, is that by Adams, Humason, and Joy,[10] repeated here for comparison with the spectrogram obtained after the 1933 outburst:

> A spectrogram of this star on July 20, 1923, shows a very strong bright $H\beta$ and a fainter emission line at $H\gamma$. Several of the prominent lines of ionized iron are present as faint emission lines. The absorption spectrum is difficult of classification, but seems to be of about type G5. The radial velocity, as determined from the bright lines, mainly of hydrogen, is approximately −35 km.

A re-examination of this spectrum confirms the probable presence of absorption lines, but the classification of the absorption spectrum as "of about type G5" is open to question. The difficulty in classifying this spectrum mentioned above rises mainly from the fact that the exposure is insufficient for the continuous spectrum, although the bright lines are well above the limit of the plate. In the region from $H\beta$ to λ 4400 the continuous spectrum is of moderate intensity, but from λ 4400 toward the violet the intensity drops rapidly and becomes exceedingly weak at λ 4150±. As the absorption lines are too weak and ill defined to measure accurately, it has been impossible to identify them with certainty; but it can be stated that if the G band or λ 4226, two of the strongest absorption features in the spectra of G5 stars, are present in this spectrum, they must be exceedingly weak. In fact, they appear to be absent. The basis for classifying the spectrum in 1923 as "of about type G5" must then have been primarily the relative intensity of the continuous spectrum rather than the absorption lines.

The spectrogram obtained on August 16, 1936, three years after the last outburst, is quite different in appearance. Seen as emission lines are λ 5007 and λ 4959, $H\beta$, $H\gamma$, and $H\delta$. The continuous spectrum is much stronger, relative to the emission lines, than on the 1923 plate and extends well into the violet. No absorption lines can be seen.

It is of interest to note that the spectrum of this star in 1923, twenty-five years after the first outburst, is the only one so far

[9] *Pub. A.S.P.*, **45**, 301, 1933. [10] *Ibid.*, **39**, 366, 1927.

observed to have absorption lines which are strong enough to be observed in small-dispersion spectra and at such a long interval after the outburst.

NOVA SAGITTARII (1919)

This nova reached a maximum of 7.0 in 1919, and at the present time is about fourteenth magnitude. A single spectrogram taken on August 17, 1936, shows a strong continuous spectrum extending well into the violet without either absorption or emission lines.

NOVA AQUILAE (1918)

Unlike the unsymmetrical envelope about Nova Persei (1901), the nebulosity surrounding Nova Aquilae appears to be uniformly distributed in a sensibly circular shell concentric with the nova. The nebulosity is gradually becoming fainter, and Baade finds on plates taken in July, 1936, that it is stronger in yellow light than in blue. The angular increase in the radius of the expanding disk is still of the same order as that found by Hubble and Duncan in 1927, 1″ per year.[11]

An attempt to photograph the spectrum of the nebulosity surrounding the nova was made in 1934, but the relative faintness of the nebulosity and its proximity to the nova prevent the nebular spectrum from showing. Only the greatly overexposed spectrum of the nova appears on the plate.

On May 30, 1938, the spectrum of the nova contained the following emission lines (intensities in parentheses): $H\beta$ (1), 4686 (1), $H\gamma$ (5), $H\delta$ (3). The continuous spectrum is strong and extends into the violet region.

NOVA LYRAE (1919)

Nova Lyrae is now one of the bluest of the old novae. Its magnitude has been estimated as 15.3, but on account of its blueness the star is very active photographically. Two spectrograms obtained on June 19 and June 20, 1936, show an extremely strong continuous spectrum with neither emission nor absorption lines observable. The intensity of the continuous spectrum in the violet is much the same as that of O-type stars.

[11] *Mt. W. Contr.*, No. 335; *Ap. J.*, **66**, 59, 1927.

NOVA SAGITTARII (1898)

The identification of this nova is uncertain, as only its position is indicated on the chart supplied by the Harvard College Observatory. The object observed as the nova agrees in position with that marked on the Harvard chart, and is the brightest star within 24″ of this position. Its magnitude has been roughly estimated as 16.5. The single spectrogram, taken on July 22, 1936, is weak on account of the southern declination of the nova, as a result of which the violet region has been much reduced by atmospheric absorption. The spectrum appears, however, to be similar to that of many of the other old novae in the fact that no lines are seen. The blue end is so weak, however, that it is not possible to state that H and K are not present. The G band is absent, however, and one certainly has the impression that the star is blue.

NOVA VULPECULAE (1670)

Several stars have been observed in this field in an attempt to locate the nova. According to the spectra, however, there is not much doubt that so far the nova has not been observed.

The first star observed was the object suspected by Steavenson of being variable in light.[12] He estimates the visual magnitude as 15.5 at present and finds no variation in the light during the past few years. Measures made at Mount Wilson give a value of 17.1 for the photographic magnitude. On fifteen direct photographs of the field, taken by van Maanen for measures of parallax and proper motion, no indications of variability can be detected. The plates cover a two-year interval between September, 1935, and September, 1937. Four spectrograms have been obtained: April 26, 1935; July 20, 1936; and two plates on August 16, 1936. The estimated spectral type is F8, and no change can be detected in the spectrum. While, on account of its spectral class, it is not likely that this star is the old nova, it seems, according to Steavenson's observations, very probable that an increase in brightness has occurred within a comparatively recent time, as the object was not noted by Barnard in 1913, although near-by fainter stars are included on his chart of the field. In addition to Steavenson's star, three others have been observed in

[12] W. H. Steavenson, *M.N.*, **95**, 78, 1934–35.

or near the field of Nova Vulpeculae. The first of these is Hind's star, or Barnard No. 26 on Steavenson's chart, given as of about the eleventh magnitude; its spectrum is G5. The second is Barnard No. 20, for which Steavenson's rough estimate of the magnitude is 13.7. Its spectral type is A0. The third star is considerably farther away from the computed position of the nova and was observed because of its remarkably red color, as seen at the Cassegrain focus of the 100-inch. This star is 1′.6 south, 10′.6 preceding Hind's star or Barnard No. 26. Its spectrum has been classified as M2, and the visual magnitude is roughly about 11.5.

It is improbable that any of the foregoing stars is the old nova, and consequently the search will be continued by observing spectroscopically several of the fainter stars in the near vicinity of the computed position.

On the chart (Pl. X) of this field Steavenson's star is indicated by an "S"; Hind's star, by the Barnard number 26; Barnard No. 20 is designated as 20; and the M-type star, by "M."

NOVA CYGNI (1920)

According to Steavenson, this nova was about 15.5 in 1936. Since 1923 five spectrograms have been obtained,[13] the two earliest

TABLE 4

EMISSION-LINE INTENSITIES IN THE SPECTRUM OF NOVA CYGNI (1920)

Date	3970	4102	4340	4363	4686	4961	4959	5007	Continuous
1923, May 7	.....	.....	2	0	2	5	60	100	Moderate
1927, June 18	.....	.....	2	0	5	5	25	60	Moderate
1934, Sept. 6	5	8	10	2	15	15	2	5	Strong
1936, June 17	2	12	15	2	12	8	0	2	Strong
1938, May 26	.....	2	2	.....	2	2	.....	.....	Strong

of which have previously been described. For comparison they are included in Table 4, which gives the intensities of the emission lines observed and the strength of the continuous spectrum for each plate. The plate of May 26, 1938, is very weak, since the exposure was stopped by clouds. It is consequently impossible to say that the

[13] Adams, Humason, and Joy, *op. cit.*, p. 365.

[*O* III] lines at λ 4959 and λ 5007 have actually disappeared completely on this date. The continuous spectrum is strong and extends well into the violet on the later plates. The two earliest plates were taken with a dispersion of 125 A/mm at $H\gamma$.

NOVA SAGITTAE (1913)

This nova is interesting on account of its parallax and proper motion. Van Maanen's value for the parallax is $+0''.013$, which gives $+2.8$ for the absolute magnitude at maximum, while for the proper motion he finds $0''.080$,[14] which is exceptionally large for stars of this class.

The nova is now of about the fifteenth magnitude, and its light remains fairly constant. Two spectrograms have been obtained, on July 11 and August 4, 1934. Both plates show a fairly strong continuous spectrum which extends well into the blue region with neither absorption nor emission lines visible. Although this nova, according to van Maanen's parallax, is a dwarf, its spectrum is in all respects similar to that of the intrinsically brighter novae.

NOVA CYGNI (1876)

According to Steavenson, this nova is still variable, with a range of 0.8 mag. around a mean of about 14.8 for the past few years. Spectrograms were obtained on August 4, 1934, and August 16, 1936. On both plates the continuous spectrum is strong and extends well into the violet. No absorption lines are seen; but on the later plate, $H\beta$, $H\gamma$, and $H\delta$, although extremely weak, are suspected of being bright.

NOVA LACERTAE (1910)

The light of this star seems to remain about constant at 14.4. Two plates obtained on July 22, 1936, show a strong continuous spectrum extending well into the violet region of the spectrum. Neither emission nor absorption lines can be seen. The star is decidedly blue and, with the exception of Nova Lyrae, is one of the bluest observed.

DISCUSSION

Besides B Cassiopeiae and Nova Vulpeculae, sixteen objects have been observed. Of these it is probable that fifteen have been cor-

[14] *Mt. W. Contr.*, No. 321, p. 5, 1926.

rectly identified and that the sixteenth, Nova Sagittarii 1898, is still doubtful. The most noticeable feature is the fact that, with the exception of T Coronae, all the objects are now decidedly blue. The extension of the continuous spectrum into the violet region corresponds to that of the early B- and O-type stars. In many of the older novae the emission lines have disappeared, or are now so nearly of the same intensity as the continuous spectrum as to be unobservable with small dispersion. A critical classification of the spectra is impossible on account of the small dispersion.

As for B Cassiopeiae and Nova Vulpeculae, it seems certain that in neither case are any of the objects observed to be identified with the old nova. The fact that the old novae that can be reliably identified are, without exception, blue stars having a strong continuous spectrum, with no absorption lines observable with small dispersion, leads one to expect that the spectra of B Cassiopeiae and Nova Vulpeculae should now be closely similar. Both novae differ greatly from the other objects in the list as regards the interval of time since the outburst—366 years for B Cassiopeiae and 268 years for Nova Vulpeculae. It should also be remembered that, since B Cassiopeiae was most probably a supernova, its spectrum at minimum may differ from that of the common galactic novae at minimum.

The enigma concerning the spectrum of T Coronae, which for years has been classified as Mb, is reasonably explained if it be conceded that the nova is a faint blue companion of the red star. Such a concession would explain the presence of the high excitation line at λ 4686 in an M-type spectrum, the difference in displacement between the bright and dark lines, and the peculiarity in the light-curve.

Densities.—Densities of novae in the final state can be obtained by using the spectral types given in Table 1 (O type), in case the distances are known. For two objects, Nova Persei (1901) and Nova Aquilae (1918), reliable values of the distance can be obtained from the observed expansions of their nebulous envelopes, by comparing the measured transverse motions (in seconds of arc per year) with the radial motions, obtained spectroscopically in kilometers per second. In the case of Nova Persei (1901), $m - M = 8.9$; and since the mean m_v is now 13.0, the present $M_v = +4.1$.

If we assume that the temperature, and hence the surface brightness, of an old nova is the same as that of a normal O-type star, we have

$$\log r - \log r_0 = 0.20\,(M_0 - M)$$

and

$$\log d - \log d_0 = 0.60(M - M_0) + \log \mathbf{M} - \log \mathbf{M}_0\,,$$

where r is the radius, M the absolute magnitude, d the density, and $\mathbf{M}$ the mass of the nova, and r_0, M_0, d_0, and $\mathbf{M}_0$ the corresponding values for a normal O-type star.

For a normal O-type star we may take

$$M_{0v} = -3.5\,, \qquad \mathbf{M}_0 = 16\,\odot, \qquad d_0 = 0.1\,\odot\,.$$

For Nova Persei we have $M_v = 4.1$; and if it was a normal star before its outburst, its mass would be about unity. Using these values, we find $d = 220\,\odot$.

For Nova Aquilae (1918), $m - M = 7.8$; m_v is now 10.8, $M_v = +3.0$. Hence, if $\mathbf{M}$ is $1.5\,\odot$, the mass corresponding to $M_v = +3.0$, then $d = 70\,\odot$.

For the remaining stars in the list, reliable distances are not known, but the mean density for the group, T Coronae, T Pyxidis, and RS Ophiuchi being omitted, can be determined by using -7.0, the mean absolute magnitude of galactic novae at maximum, together with $\overline{M}_{max} - \overline{M}_{min} = +9.8$, whence

$$\overline{M}_{min} = +2.8\,, \quad d = 60\,\odot\,.$$

It appears, therefore, that the densities of old novae are intermediate between those of ordinary stars and of the white dwarfs, which are of the order of 10^4 times that of the sun. Milne's suggestion that the nova process is a transition from a star of ordinary density to a white dwarf[15] is, therefore, not entirely substantiated by these observations.

It is our intention at Mount Wilson to reobserve many of the novae included in the present list with larger dispersion in order

[15] *M.N.*, **91**, 4, 1930; *Obs.*, **54**, 140, 1931.

that a more critical classification of their spectra may be made. In addition, we hope in the near future to obtain a better value of their present apparent brightness, and finally a more reliable determination of the densities.

I wish to thank Dr. Shapley, director of Harvard College Observatory, for his kindness in forwarding charts of six fields from which identifications have been made.

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PROMINENCE STUDIES*

ROBERT R. McMATH AND EDISON PETTIT

ABSTRACT

Eruptive prominences.—Data for seven new eruptive prominences confirm the laws of motion previously stated. The end velocity is now shown to revert occasionally to a whole multiple of the second preceding velocity, which explains some apparent exceptions to the second law. One prominence reached the unparalleled height of 1,550,000 km and another the record velocity of 728 km/sec. The former was near the north pole of the sun.

The dilemma of light-pressure.—A calculation of the Milne effect shows that we cannot assume that light-pressure balances gravity at the sun's surface. Velocities of more than 100 km/sec are developed in a few minutes from an initial 10 km/sec velocity. Calcium tends to separate widely from hydrogen.

Prominences of class III.—The motions in the fountain-like loops of class III*b* prominences are down both branches from the ends of the bright tops of the arches. Practically all obey the first law, and about 20 per cent the second law of prominence motion; 7 cases show no change in velocity.

Ejections.—Small faint lumps of chromospheric matter are sometimes ejected from a spot area, often at considerable angles to the vertical. These ejections seem to move in nearly straight trajectories and do not return.

Centers of attraction.—Examination of the dissolution of an active prominence projected on the disk shows that centers of attraction are not visible as surface markings. With one exception, prominence streamers have always been seen to enter centers of attraction from one side only. Fifty per cent of all the streamers from class I prominences measured followed both laws of prominence motion.

Coronal prominences.—When a center of attraction becomes very active, long streamers not connected with the active prominence enter the center of attraction from great heights. These seem to come from space in the middle or outer corona. They obey both laws of motion of eruptive prominences.

The study of prominences by the motion-picture method at the 50-foot tower telescope of the McMath-Hulbert Observatory was described in a previous paper.[1] These observations have been continued during the past season with some improvements in equipment. The two off-axis mirrors originally designed for the tower are now in regular use, one having a focal length of 40 feet, the other of 20 feet. A Cassegrain convex is now provided, which, substituted for the $9\frac{1}{2}$-inch flat in the convergent beam of the 40-foot focus mirror, increases the focal length to 50 feet. This arrangement is used to study details in small chromospheric phenomena. A Ross lens of 74 inches focal length is now provided for routine examination of

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 597.

[1] *Mt. Wilson Contr.*, No. 568; *Ap. J.*, **85**, 279, 1937.

the whole sun and for extremely high eruptive prominences. The focal lengths now available are 50, 40, 20, 8.1, and 6.17 feet.

The 40- and 20-foot focus mirrors are now mounted on the same bracket, so that, by a partial rotation of the mounting and a small linear displacement upon it, the two can be interchanged without moving the coelostat, thus leaving the orientation of the image fixed upon the spectrograph. The same thing is accomplished with the two lens systems by substituting a plane mirror for the concaves upon the same bracket. This arrangement has very much reduced the time required to change the image scale when a prominence moves out of the motion-picture frame.

A Herschel eyepiece and a right-angle prism are mounted on a swivel, upon which they can be moved into the beam at a fixed distance above the first slit, thus providing precise focusing. Neutral-tint glass of 10 per cent transmission has replaced the usual occulting disks, thereby giving a record of the chromosphere, as well as of the prominence being photographed.

The measurements given in the present paper were made with a standard motion-picture projector (Holmes), arranged to move the film in either direction by means of electric fast or slow motion or by hand, frame by frame. A frame counter was also supplied. This apparatus projects an image of a frame 23×17 inches upon a vertical opal glass screen, which can be moved in rectangular co-ordinates to adjust the image. The scale used varies from 400 km/mm for the 50-foot image to 3240 km/mm for the 74-inch. The major part of the work was done with the 40-foot focus mirror, scale 500 km/mm. Illumination was provided with a 100-watt lamp and concave mirror. This lamp could be used with the stationary film without danger of ignition, the film becoming just sensibly warm with prolonged exposure. In the latter part of the work it was found that considerable improvement in measurement could be made if the point to be set upon were marked with ink upon the opal glass screen. This procedure avoids the inaccuracy due to loss of the setting-point while the scale is being adjusted, which is especially important when the objects are large or very faint. The scale was established by inserting a strip of film with centimeter marks ruled upon it with a diamond ruling-machine; the projected magnification was measured

with the same celluloid scale as was used in measuring the prominences. In this operation the apparent solar semidiameter was increased by 4500 km to correct for the height of the chromosphere, a value obtained from a number of K spectroheliograms taken at Mount Wilson, having the solar image at λ 0.4 μ impressed upon the same plate. Of course this correction depends upon the image density, but its variability can affect the result only to a second-order degree. A precision slit-drive worm has eliminated the *réseau* on the images, which facilitates measurement.

ERUPTIVE PROMINENCES

Two laws have been found to characterize the motions of eruptive prominences:[2] (1) the speed is uniform except that at intervals it increases suddenly; (2) with very few exceptions, the speed following such a change is a small integral multiple of the value preceding the change.

These laws of prominence motion are still not satisfactorily explained, and any new observations are very desirable. A problem of first importance involves the accelerations which take place when a change in velocity occurs. The older observations seemed to show that the change in velocity occurred in less than 5 or 10 minutes, and our observations of 1937 now reduce this time element to less than 1 minute.

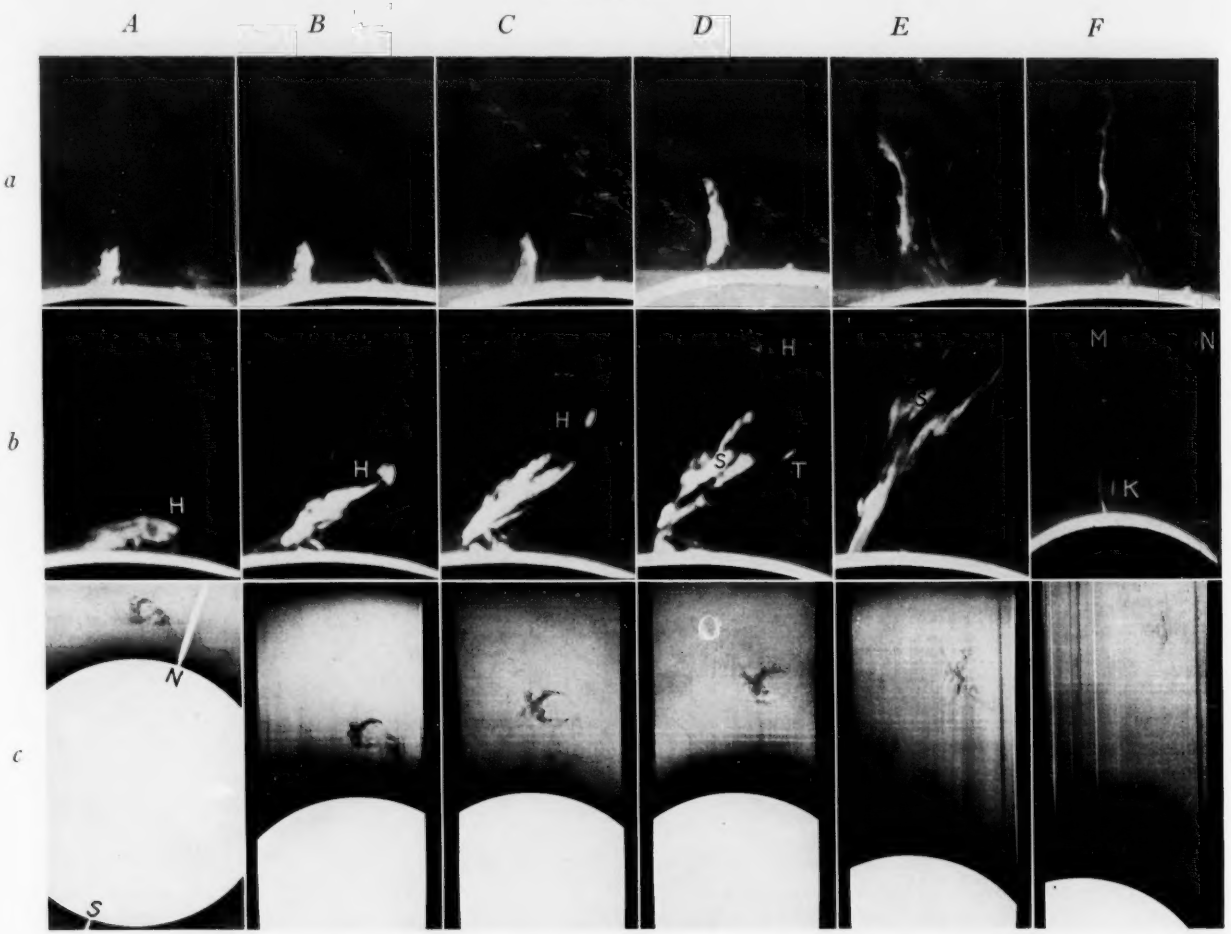
While eruptive prominences usually move in a direction approximately radial to the sun, the motion is sometimes at a rather large angle to the radius. When a considerable deviation does occur, the trajectory of the prominence becomes a curved line, concave toward the chromosphere; as the prominence rises, it still pours streamers upon the chromosphere to a center of attraction, even in preference to a near-by sunspot, and this is the direction which the deviation from radial motion takes.

The surges, a distinct subclass of the sunspot type, and the ejections, which will be described later, are the only prominences which we observe to rise directly from the chromosphere. The surges also have the characteristic of returning to the chromosphere along the same path, while the ejections leave the chromosphere entirely.

[2] *Mt. W. Contr.*, No. 552; *Ap. J.*, **84**, 319, 1936.

UNIV. OF WICH.

PLATE XI



ERUPTIVE PROMINENCES

a) Lake Angelus, July 18, 1937. *A–F*, G.C.T. 14^h00^m, 15^h42^m, 18^h12^m, 20^h16^m, 21^h19^m, 21^h43^m. Dark-glass occulter used in *D*. *b*) Lake Angelus, September 17, 1937. *A–F*, G.C.T. 14^h51^m, 15^h03^m, 15^h06^m, 15^h09^m, 15^h14^m, 16^h06^m. In *D* and *E* the velocity of H is 728 km/sec. Prominence leaves picture at 1,000,000 km in *F*. *c*) Mount Wilson, March 20, 1938. *A–F*, G.C.T. 17^h12^m, 17^h35^m, 18^h01^m, 18^h18^m, 19^h03^m, 19^h31^m. *A* near N pole; *F* shows prominence 1.1 solar diameters high.

Eruptive prominences, on the other hand, appear first as active or class III*c* prominences, which, after a period varying from some hours to weeks or months, suddenly rise and move away from the sun.

So far we have not distinguished between eruptive prominences connected with sunspots and those connected with centers of attraction; the nature of the motion is the same. Since centers of attraction and eruptive prominences are found even near the poles of the sun, we conclude that the magnetic field of the spot does not enter the problem of the motive force.

All parts of an eruptive prominence do not move with the same velocity; and sometimes sections move with velocities radically different from that of the main body. These and other features will be seen in the cases that follow. For convenience we identify the prominences by number in continuation of the catalogue already given.[1]

Number 41 was reported by M. G. Fracastoro.[3] The eruption occurred on May 29, 1937, in solar latitude N. 70° on the east limb, and therefore no spot was associated with it. The data are the heights observed visually by himself at Arcetri and by W. Brunner at Zürich. Plot 41A, Figure 1, is from measures made by Fracastoro on drawings by Brunner.

Number 42 was observed at Lake Angelus on July 15, 1937, on the east limb in solar latitude N. 20° over a single growing spot, Mt. W. 5472. The trajectory was a slightly curved line, inclined 42° to the radius. Clouds obscured the concluding features of the eruption. A surge formed at the same point.

Numbers 43, 44, and 45 (Pl. XI*a*) were observed at Lake Angelus, July 18, 1937, on the east limb in latitudes S. 50°, S. 35°, and S. 30°, respectively, and were all in motion at the same time. A small spot-group developing in latitude S. 22° seemed to play no part in the eruptions. Number 43 began as a very low, faint prominence which rose slowly in a trajectory concave to the chromosphere where a center of attraction was located, the path making an angle of 40° to a radius. Owing to reduced visibility, probably haze, the point of aim was twice lost, which accounts for the discontinuities in the plot in Figure 1 and also the large deviations from the mean line.

[3] *Rend. Accad. nationale dei Lincei* (*cl. fis.*), Ser. 6, **25**, Part 12, 1937.

In *D*: *b*) Lake Angelus, September 17, 1937. *A–F*, G.C.T. 14^{h}51^m, 15^{h}03^m, 15^{h}06^m, 15^{h}09^m, 15^{h}14^m, 16^{h}06^m. In *D* and *E* the velocity of H is 728 km/sec. Prominence leaves picture at 1,020,000 km in *F*. *c*) Mount Wilson, March 20, 1938. *A–F*, G.C.T. 17^{h}12^m, 17^{h}35^m, 18^{h}01^m, 18^{h}18^m, 19^{h}03^m, 19^{h}31^m. *A* near N pole; *F* shows prominence 1.1 solar diameters high.

Number 45 was a spearhead-shaped prominence, probably as bright as the chromosphere at the limb, which moved in a direction

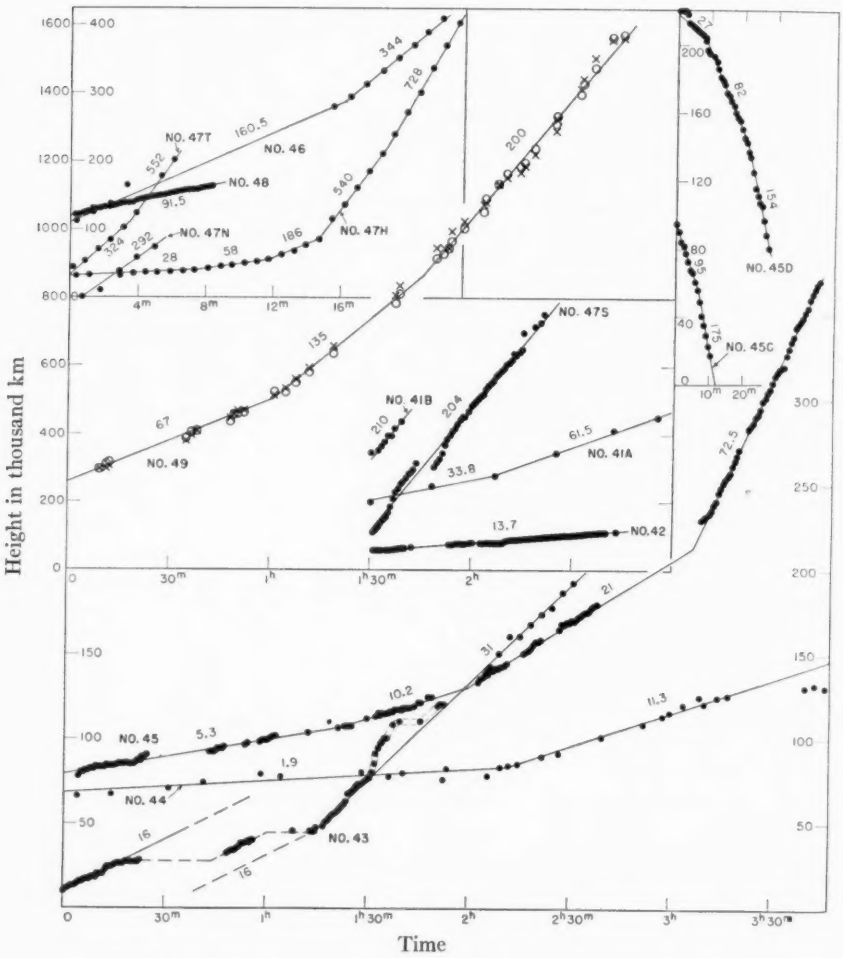


FIG. 1.—Time-height diagrams of eruptive prominences, 1937–1938. Identification by catalogue number. 45C and 45D, streamers returning to chromosphere from eruptive prominence of July 18, 1937. Velocities in km/sec are indicated above the straight-line sections in this and succeeding figures.

radial to the sun. At the beginning of the 5.3-km/sec phase, the possible influence of any Doppler effect on the appearance of the prominence was tested by setting the K line alternately 1 A to the

red, at the center, and 1 A to the violet on successive exposures over a period of 1 hour. No appreciable effect on the prominence-form was noticeable.

Throughout the eruption a large sheaf of streamers returned from the prominence to a center of attraction 5° to the south in preference to the sunspot-groups Mt. W. 5473 and 5474, 13° to the north. The velocities attained by these streamers were 150–160 km/sec (plots 45C and 45D, Fig. 1). It would seem, then, that these three prominences were not associated with the sunspot-groups.

Number 46 was observed at Lake Angelus on September 1, 1937. Taken under exceptionally difficult circumstances through cloudy sky, only twenty-eight frames are available, but they suffice to indicate the velocities. Time was not available even for identification of position.

Number 47, photographed by H. Sawyer and J. Brodie at Lake Angelus[4] on September 17, 1937, reached a record height of 1,000,000 km and a velocity of 728 km/sec. This prominence was one of a number observed over the great sunspot-group which first came over the east limb in latitude N. 31° on July 22, 1937, one of the five largest spot-groups ever photographed. When it appeared on the east limb on September 16, it had decreased in size; but the field-strength was still probably about 3000 gausses, a value found a few days later. On the following day a rather bright prominence of class III*c* appeared over this spot; the first exposure at Lake Angelus was made at 14^h10^m G.C.T. At 14^h47^m it began to rise, and 1^h43^m later left the frame of the motion-picture camera at an elevation of 1,000,000 km.

Plate XI*b* shows six stages of this eruption. In *A* the head H is in a state of great turmoil, as if an uprushing wind were forcing it to rise. In *B* the head has separated; and in *C* and *D* it can be identified, floating away, followed by the body of the prominence at a lower speed. The bright cloud S could be identified over a considerable part of the scene and furnished a velocity of the body of the prominence. In *F*, where the focal length of the objective of the tower telescope has been reduced from 20 to 6.17 feet, the bright

[4] *Pub. A.S.P.*, **49**, 305, 1937.

clouds M and N are seen leaving the frame of the motion-picture camera at an elevation of 1,000,000 km above the chromosphere.

Figure 1 shows the time-height plots of the clouds H and S. The velocities of H are 28, 58, 186, 540, and 728 km/sec, while S exhibits only one velocity, 204 km/sec, and could be followed to a height of 750,000 km along a straight trajectory at an angle of only 6° to the solar radius. There is a little evidence that the velocity of S may have increased to 400 km/sec, but this depends on only three observations and has little weight. The deviations of the measurements from the mean line at the beginning of the plot are probably due to changes in the form of the cloud S.

The cloud H was projected at an angle of 15° to the solar radius in a practically rectilinear trajectory, probably the resultant of the small angle and high velocities involved. During the 728-km/sec phase, the velocity was so high that the cloud moved over its own length during the exposures. It must therefore have trailed this distance on the film, since in our equipment the exposure is nearly continuous ($\frac{1}{2}$-sec elements). It follows that this cloud, as it appears in Plate XI, could have been formed by the trailing of a thin, bright cloud approximately perpendicular to the trajectory.

Such a case was the bright cloud T in *D*, which was moving almost endwise to the right, however; had it moved upward, it probably would have looked like H. Cloud T moved in a straight line which, projected backward, is nearly tangent to the sun where it enters the frame on the left. This is the neck of the head H, and its trajectory makes an angle of 42° with that of H. The plot in Figure 1 is that of the faint end of T, which persists unchanged longer than the bright end and indicates two velocities, 324 and 552 km/sec.

Both H and T reached velocities exceeding the velocity of escape from the sun at the observed elevation. So far, however, this fact has no great significance, since uniform motion, however small, will carry the prominence indefinitely into space.

These ejections of very rapidly moving isolated masses of gas from a prominence moving at a slower speed we shall call *secondary ejections* or *secondaries*. They may also occur in a stationary cloud, such as the class III*c* prominence of March, 1920,[5] or in a surge, as will be seen.

[5] *Pub. Yerkes Obs.*, 3, Part 4, 1925, Pl. XXXV.

The exposures on the *Hα* disk taken at Mount Wilson at intervals of $3^{m}.35$ show at the beginning of the eruption, at $14^{h}58^{m}.98$ or $15^{h}02^{m}.33$, only a faint flocculus beneath the prominence and over the spot; but at $15^{h}05^{m}.33$, about the time the velocity of H became 728 km/sec, this flocculus suddenly became very bright, probably a magnitude brighter than the chromosphere, remained bright for half an hour, then gradually faded to its original intensity.

The brightness of the prominence at the beginning of the eruption was about equal to that of the chromosphere at the limb in K2, but "noticeably brighter" in *Hα*.[6] Toward $17^{h}25^{m}$ clouds intervened and when observations were resumed at $18^{h}35^{m}.11$ a bright class III*b* fountain was found over the spot. This phenomenon was observed in the case of the great surge of August 8, 1936, previously reported.[1] The details of motion in this phase of the prominence will be given later.

An examination of Figure 1 shows that the time interval required by this prominence to change its velocity at the three breaks in the plot must have been less than the interval of 45 seconds between observations and was probably less than about 10 seconds. While we cannot yet say how much smaller this time interval may be, it now begins to appear that the change in velocity is not one of the bulk but of the individual atoms, and that all motions which appear in bulk are really a streaming of individual atoms. Tests were made for effects of any Doppler displacements on the form of the prominence or on its details at the time when S had risen 311,000 km. No effect could be detected.[6]

Like No. 44, this prominence sent streamers back to the chromosphere, but only in the last stages. In Plate XI*b*, the long streamer K is returning to the chromosphere with a velocity of 205 km/sec, practically the same as the velocity of rise of *S*. The velocity and direction of motion of parallel streamers cannot surely be determined on account of rapid fading.

The cloud N could be followed through five exposures as it left the frame of the motion-picture camera at a height of 1,000,000 km. The resulting plot (Fig. 1) shows a uniform motion of 292 km/sec. The trajectory projected upon the chromosphere comes to nearly the same point as that for T, although this may be a coincidence.

[6] Observed by H. Sawyer at the 10½-inch telescope.

Number 48 was photographed at Lake Angelus on September 26, 1937. It had the form of a giant arch which rose and broke apart, while matter was fed down both branches to the chromosphere, much like the eruptive prominence of July 15, 1919, observed at the Yerkes Observatory. Both these prominences were associated with spot-groups—No. 48, for example, with a complicated group (Mt. W. 5578) on the east limb of the sun in latitude N. 9°, into which the north branch of the arch was pouring.

This prominence was photographed in *Hα* with both slits 0.4 mm wide, so that a radial velocity of ±90 km/sec would be required to remove the line from the slit. With these wide slits the appearance of the *Hα* spectroheliograms is much like those taken in K.

Number 49, observed by J. O. Hickox at Mount Wilson on March 20, 1938, appeared as a loop, not very bright, already 296,000 km above the chromosphere when first observed at 17^h10^m G.C.T. (Pl. XI*c*). In the succeeding 2^h30^m it rose to the unprecedented height of 1,550,000 km from a point in solar latitude N. 75° on the west limb, pouring into a center of attraction in latitude N. 85°. The trajectory along which the prominence moved 1,566,000 km was nearly a straight line inclined at an angle of 25° to the radius toward the center of attraction.

The observed velocities V_0 are collected in Table 1. Where more than one velocity was found, the ratio F of each velocity to that preceding is given. It is seen that F is quite well represented by small whole numbers, save in the last velocities (starred) of Nos. 45, 47H, and 49. We note, however, that these last velocities are in every case whole multiples (italics under "F") of the second preceding velocity (italics under "V_0"). The same phenomenon is also seen in the eruptions of August 6, 1931,[7] and May 26, 1916,[8] shown at the end of the table. We are now able to explain by this reversion principle some outstanding exceptions to the second law previously observed.

In all these cases the observations began when the prominence was already at a high elevation, 200,000–300,000 km, above the chromosphere, and only two velocities were found. We may suppose, then, that a velocity change occurred before the observations

[7] *Mt. W. Contr.*, No. 451; *Ap. J.*, **76**, 9, 1932.

[8] *Bull. Kodaikanal Obs.*, **3**, No. 55, 1916.

began and that the first observed velocity is a small whole multiple of it; the second, which is also the last, must then also be a multiple of it. These hypothetical velocities are given in parentheses in Table 2; the values of F are, of course, also hypothetical.

TABLE 1

No.	Date	Spot?	V_0	F
	1937			
41A	May 29	No spot	33.8, 61.5	2
41B	29	No spot	210	.....
42	July 15	Spot	13.7	.....
43	18	Spot	16, 31	2
44	18	Spot	1.9, 11.3	6
45	18	Spot	1.7, 5.3, *10.2*, 21.0, 72.5*	3, 2, 2, 7
46	Sept. 1	?	160, 344	2
47H	17	Spot	28, 58, *186*, 540, 728*	2, 3, 3, *4*
47S	17	Spot	204	.....
47T	17	Spot	324, 552	2
47N	17	Spot	292	.....
48	26	Spot	91.5	.....
	1938			
49	Mar. 20	No spot	*67*, 135, 200*	2, *3*
	1931			
35	Aug. 6	No spot	5, *19*, 74, 126*	4, 4, *7*
	1916			
9	May 26	No spot	*64.2*, 191.7, 316.6*	3, *5*

TABLE 2

No.	Date	V_0	F
2	1919, July 15	(18.2), 37.0, 163.9	(2), (9)
30	1924, June 23	(14.5), 43, 73	(3), (5)
28	1928, Nov. 19	(40), 81, 200	(2), (5)

There seems to be strong evidence, then, that a possible reversion of the end velocity to a multiple of the second preceding velocity should be included in the second law of prominence motion.

NATURE OF THE IMPELLING FORCE

Aside from the first and second laws of prominence motion, we can make a few additional generalizations which any theory of these motions must cover. Most of this information comes from a study

of the time-height plots of Nos. 1, 22, 31, 35, 45, and 47, in which more than two changes in velocity were observed.

a) The changes in the value of the force come at intervals which, though irregular, generally become shorter until the last impulse, which shows some tendency to increase. Table 3 gives the duration of each velocity occurring in the time-height diagrams when several velocity increases occur.

b) The number of changes in velocity for any one eruption seems to be nearly independent of the height to which the prominence rises and of its duration. For example, the duration of No. 1 (Table 3) was several hours, while that of 47H was only a few minutes.

TABLE 3

DURATION OF SUCCESSIVELY INCREASING VELOCITIES IN ERUPTIVE PROMINENCES

No.	Duration (Min.)	Total Rise (1000 Km)	Final Height (1000 Km)
1	154, 132, 48, 63+	490	760
22	120, 90, 40, 30, 6+	288	382
35	60, 75, 23, 35+	571	620
47H	8, 4.8, 3.1, 3.7, 4.6+	370	400

c) The changes in velocity are not always the same for all parts of a prominence nor in exactly the same direction. In fact, some parts act quite independently. Probably the most extreme example is No. 47H. The head H (Pl. XI) showed four changes in velocity (Fig. 1, plot 47H) in a rise of 370,000 km, while a part of the body, S (Fig. 1, plot 47S) moved 650,000 km without change in velocity and at a speed slower than the final speed of H. T moved at a considerable angle to both H and S.

d) The motions are often at considerable angles to the radius. The greatest deviation so far was 50° for No. 3, although 25°–40° is common.

e) There is but little correspondence between the initial surface brightness of the prominence and the final velocity it attains. While only very rough data are available, we may rate the spectroheliographic results by comparison with the chromosphere at the limb and with any other prominence near by. The most rapidly moving

prominence yet observed was No. 47H, which reached 728 km/sec. Its brightness exceeded that of the others and was about equal to the brightness of the chromosphere at the limb. The next highest velocity is 344 km/sec for No. 46, which we estimate to be about 0.4 the brightness of the chromosphere, or a little less. Number 9 reached 317 km/sec, but no brightness data are available to us. Numbers 28, 49, and 11 reached 200 km/sec, but data for the latter are also unavailable. Numbers 28 and 49, however, were about as bright as the ordinary prominences, i.e., about one-sixth the chromosphere. Numbers 2, 25, 35, 37, and 40 exceeded 100 km/sec. None of these was sensibly brighter than the ordinary prominences. For No. 35 we have records from a height of 47,000 km and, even at this low elevation, its intensity was about that of other ordinary prominences on the limb.

It seems, then, that extraordinary brightness is not encountered in eruptive prominences, at least not until the velocities exceed 300 or 400 km/sec. The highest prominence on record, No. 49, had the intensity of ordinary prominences, one-sixth the chromosphere in the early stages of its rise to an altitude of 1.12 solar diameters, although the beginning was not observed.

f) The two laws of prominence motion seem also to apply to knots and streamers pulled into the chromosphere from active and sunspot-type prominences. The coronal prominences and streamers returning to the chromosphere from the eruptive prominences themselves (Fig. 1) also obey these laws.

g) We have some evidence, which depends principally on Nos. 25 and 37, that one part of a prominence may change its velocity while another part, moving with another (in each case higher) velocity, continues unchanged.

Considerations (*c*), (*e*), (*f*), and (*g*) conspire to eliminate light-pressure as the motive force for prominences, even if we use it in intermittent form. We will now consider various theoretical discussions of prominence motion.

THE DILEMMA OF LIGHT-PRESSURE

There can be no value in considering a prominence as moving with continually accelerated motion, as M. Notuki[9] has lately done,

[9] *Proc. Phys. Math. Soc. Japan*, **18**, 598, 1936.

following the precedent set by previous theories whose only salient observational fact is that eruptive prominence velocities increase with height. It is our experience that if sufficient observations were made to determine the character of the motion, the two laws always apply.

K. O. Kiepenheuer[10] has recently attempted to satisfy for the first time the actual observational facts. First, considering the magnetic moment in a sunspot, he arrives at a formula for velocity in a streamer or knot of a class III prominence which indicates uniform motion. Considering eruptive prominences, he finds that radiation pressure derived from momentary bright chromospheric eruptions in Lyman α would give sufficient velocity if a factor 10^9 times brighter than the photosphere over the spectral range of the line were allowed. This assumes that ordinary Lyman α intensity plus general line absorption of the prominence just supports it above the chromosphere. He supposes that bright chromospheric eruptions of 10 minutes' duration at intervals of an hour or so would produce the observed increases in velocity; these velocities would be otherwise constant in a gravitational field nullified by light-pressure. Resistance of the coronal atmosphere is considered but plays no part in the theory.

Now there are many points of variance between observational data and this theory. (1) We have shown that the magnetic field of a spot plays little part in the motion of streamers from a class III prominence. (2) Eruptive prominences are only occasionally found connected with sunspots; three (Nos. 10, 41, and 49) occurred within a few degrees of the poles. Bright chromospheric eruptions, on the other hand, are always connected with sunspots, save two or three doubtful cases, which were in sunspot zones, however; they are not found near the poles. (3) The duration[11] of the bright chromospheric eruptions, 12–28 minutes, would produce very noticeable curvature at the breaks in the time-height diagrams. We have now reduced the possible time interval of velocity change from 10 minutes for the former observations to less than 1 minute in the case of No. 47H (Fig. 1). (4) The resisting effect of the inner corona cannot be neglected if its density is a considerable fraction of that of the promi-

[10] *Zs. f. Ap.*, **15**, 53, 1937. [11] R. S. Richardson, *Pub. A.S.P.*, **49**, 236, 1937.

nences. It has already been pointed out[7] that prominence density is about 2×10^{13} H atoms per cubic centimeter, or nearly one-millionth atmosphere. The mean free path of the atoms is, then, one million times greater than that at terrestrial atmospheric pressure (of the order 16 cm for hydrogen). It is clear, then, that bright chromospheric eruptions cannot furnish the motive power for eruptive prominences; neither this nor any other known repulsive force acts in the quantized manner demanded by the second law.

A. Unsöld has discussed the light-pressure theory of prominence motion[12] in an excellent summary. He shows that for the center of the line the ratio μ of light-force to gravitation is

$$\mu = \frac{1}{r} \cdot \frac{2\pi^2 e^2 h}{mc\lambda^3 Mg} \cdot f \cdot e^{-c_2/KT}, \qquad (1)$$

where $1/r$ is the residual central intensity of the line, $f(\approx 1)$ the line oscillator strength, and M the atomic weight of the atom considered.

Introducing into (1) $\lambda = 3950$ A, $f \approx 1$, $M = 40$, $T = 5740°$, and $1/r = 0.063$ for the H and K lines, he obtains $\mu = 1$, i.e., exact balance of a Ca^+ ion with solar gravity in the chromosphere. As the calcium component is only about 2 per cent of the mass of a prominence, and that mostly Ca^{++}, Unsöld's theory makes use of the Lyman emission by the sun and finds the light-force to be 6.6×10^{-7} gravity. This disparity is explained as a deviation from thermal equilibrium in prominences such that in (1), for the function $e^{-c_2/KT} \approx 10^{-13.5}$, we must substitute 10^{-8} in order to make $\mu \approx 1$. This is about the status of the theory of prominence motion as it stands today.

Suppose we admit that light-force does balance gravity in the chromosphere and prominences, then the Milne effect will certainly unbalance it and cause the prominences and chromosphere alike to disappear, for random motions of 10–15 km/sec are common in considerable masses of chromospheric matter and quiescent prominences.

Milne proposed[13] that, as a prominence rises, the center of the absorbing line shifts by Doppler effect to the violet in the radiation

[12] *Physik der Sternatmosphären*, pp. 422 ff., 1938. [13] *M.N.*, **86**, 459, 1926.

being absorbed and, therefore, onto the continuous solar spectrum which is much more intense and exerts greater light-force. No theory of prominence motions has so far taken account of this effect, which is really of major importance if we attempt to balance light-force against gravity at the sun's surface. Milne's discussion covers the subject of the limiting velocities to be expected in the high-speed atoms so produced; but let us examine the character of this action in detail. We will first develop the equations for a V-shaped Ca^+ line leading to a time-height diagram for comparison with the observations and then treat the $H\alpha$ line with the same theory.

A. D. Thackeray[14] and M. Minnaert have given the energy distribution over the K line obtained by photographic photometry. From energy-curves across the H and K lines taken with the 21-foot concave-grating monochromator and photoelectric amplifier at Mount Wilson, it is apparent that there is little distinction between the profiles of the two lines, and we may therefore treat both as if combined into the K line as given by Thackeray and by Minnaert. We find that if I_0 is the minimum intensity (effectively 0.1 photospheric) in the H or K line, then the intensity I in the wing at any point is given by the empirical formula

$$I = \frac{I_0 R^2}{(R+s)^2}\left[\frac{V}{100} + 1\right], \quad 0 < V < 500 \text{ km/sec}, \quad (2)$$

where V is the velocity in km/sec, R the radius of the sun in kilometers, s the height above the sun in kilometers, and where the time element t of V is in seconds. If gravity is balanced by light-force at the center of the line, the acceleration A, in terms of gravity g_0, is

$$A = g_0 \frac{VR^2}{100(R+s)^2}. \quad (3)$$

If for the constants we substitute

$$a = \frac{g_0 R^2}{100} = 1911R, \quad (4)$$

[14] *Ibid.*, **95**, 297, 1935.

we may express (3) by the differential equation

$$\frac{d^2s}{dt^2} = \frac{a}{(R+s)^2}\frac{ds}{dt}, \tag{5}$$

and, by substitution and integration, get

$$\frac{ds}{dt} = -\frac{a}{R+s} + c = V. \tag{6}$$

We may separate the variables in (6) as follows:

$$\frac{ds}{c - \frac{a}{R+s}} = dt = \frac{(R+s)ds}{(cR-a)+cs}. \tag{7}$$

Now substitute for the constants

$$(cR - a) = c_1; \tag{8}$$

then, by integration,

$$t = \frac{a}{c^2}\log_e (c_1 + cs) + \frac{s}{c} + \frac{c_1}{c^2} + c_2. \tag{9}$$

To determine the constants of integration in (9), place the prominence at the chromosphere with an initial upward velocity $V_0 = 10$ km/sec, when $t = 0 = s$; then, from (6), (4), (8), and (9)

$$c = 1921, \tag{10}$$

$$c_1 = 10R = 7 \times 10^6, \tag{11}$$

$$c_2 = 5717. \tag{12}$$

It follows from (6) and (10) that with $V_0 = 10$ km/sec the velocity will not exceed 1921 km/sec.

Inserting these constants into (9) and (5) and adapting the equation to common logarithms, we have

$$t = 835 \log_{10} [7 \times 10^6 + 1921s] + \frac{s}{1921} - 5717, \tag{13}$$

$$V = 1921 - \frac{1911 \times 7 \times 10^5}{7 \times 10^5 + s}. \tag{14}$$

It will be noted that (9) and (6) are general equations, limited only by the time corresponding to an upper velocity of about 500 km/sec, beyond which (3) does not hold. Equations (13) and (14) are also specialized by the assumption of an initial upward velocity of 10 km/sec. We now assign values to s in these equations and compute t and V; the results are shown in Table 4 in the columns headed "Ca^+". Here "V Comp." is obtained from (14); "V Obs., Av." represents the mean velocities observed at the corresponding elevations s; and the last column shows the values of V Obs. for the fastest-moving eruptive prominence so far observed, No. 47H.

TABLE 4

t	s		Δs	V Comp.		V Obs.	
	Ca^+	$H\alpha$	$H\alpha - Ca^+$	Ca^+	$H\alpha$	Av.	No. 47*H*
sec.	10^3 km	10^3 km	10^3 km	km/sec	km/sec	km/sec	km/sec
482	10	16	+ 6	37	70	.....	.....
996	50	64	+ 14	138	118	5	58
1265	100	100	0	259	152	25	186 (540)
1561	200	150	− 50	435	194	70	540
1759	300	191	−109	538	225	115	728

Since in the latter case the velocity changes, just below 100,000 km, both velocities are included.

The data in the first two columns of Table 4 are used to plot the time-height theoretical curve for Ca^+ in Figure 1*a* on the scale of No. 47H. The comparative features indicate at once that we have here no explanation of prominence motion.

Velocities determined by drawing tangents to the curve in Figure 1*a* agree very accurately with those obtained from equation (14), thus verifying both the calculus and the arithmetical work between equations (5) and (14).

As the absorption lines of the prominence pass over the wings of the H and K lines, they will encounter other absorption lines which would reduce the velocity of ascent momentarily; but, since two conspicuous lines are not likely to be encountered simultaneously in both H and K wings, the effect will be smoothed out.

We can treat the *Hα* line in the same way we have treated H and K, although not quite so directly. The narrow core of *Hα*, nearly 2 A wide in the photosphere, with wings extending several angstroms, is superposed upon the narrower prominence absorption line without wings but of unknown absolute dimensions. From the emission line, however, we judge its width must be of the order 1 A. Ordinary thermal Doppler widening will account for most of this width.

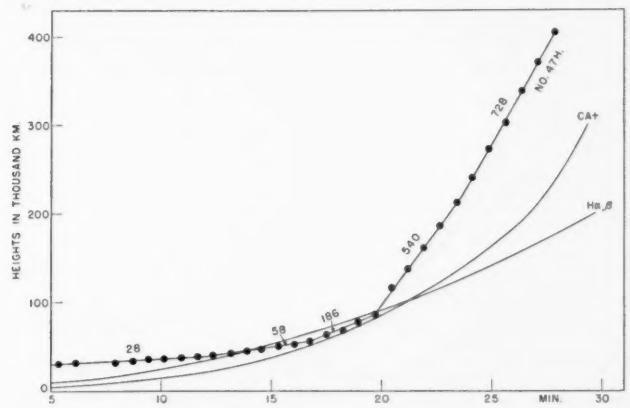


FIG. 1*a*.—Theoretical time-height diagram computed for the Milne effect in a gravitational field balanced by light-pressure for ionized calcium and neutral hydrogen with an initial velocity of 10 km/sec, compared with diagram of observations for eruptive prominence No. 47H of September 17, 1937.

We have therefore assumed that the shape of the absorption line in prominences is that of an error-curve with a half width of 1 A and a base width of 2 A. The curve was divided into nine intensity ordinates, which were multiplied into the corresponding ordinates of the photospheric line as given by Thackeray[15] for various velocity displacements and the resulting vector-curve drawn. This resembles Thackeray's curve, except in the core, which is much flattened out and V-shaped, with a minimum intensity of 0.28, the sides joining the wings at ± 1.3 A; the wings are practically identical with Thackeray's original curve.

[15] *Ibid.*, p. 296.

Considering first the V-shaped core, we have an equation like (3),

$$A = g_0 \frac{VR^2}{37(R+s)^2}, \qquad 0 < V < 60. \tag{15}$$

The constants for (6) and (9) become

$$\left.\begin{aligned} a &= 5170R, \quad c_1 = 7 \times 10^6, \\ c &= 5180, \quad c_2 = 2126, \end{aligned}\right\} \tag{16}$$

where, as for Ca^+, we have assumed an initial upward velocity of 10 km/sec. From (6) and (9) we find that V will reach its upper limit of 60 km/sec, within which (15) applies, in 245 seconds, when the $H\alpha$ prominence will be only 6869 km high.

At this point a new set of conditions exists. Equation (2) becomes

$$I = \frac{I_0 R^2}{(R+s)^2}\left[\frac{V}{380} + 1\right]. \tag{17}$$

Now $V = 60$, $s = 6869$, and the acceleration is that which the prominence has accumulated in 245 seconds, which can be computed by the substitution of these constants into (15), giving

$$A_0 = 0.434 \text{ km/sec}^2 = 1.59 g_0. \tag{18}$$

Then for a in (4) and for the integration and substitution constants we find

$$\left.\begin{aligned} a &= \frac{0.434 \times 7 \times 10^5}{380} R = 799.5R, \\ c &= 852, \\ c_1 &= 3.745 \times 10^7, \\ c_2 &= -13{,}277. \end{aligned}\right\} \tag{19}$$

In determining c_2 the foregoing conditions were put into (9), which, in turn, was used to calculate the relation of t and s.

The results for $H\alpha$ are given in Table 4, along with those for Ca^+ and the time-height curve in Figure 1*a*. We have here considered only $H\alpha$. $H\beta$ is nearly identical, and the forms of the other principal

Balmer lines are similar, though shallower; hence, the general result will be the same. It will be noted that hydrogen and calcium reach the 100,000-km level at the same time, but that previously the hydrogen is higher than the calcium, while subsequently it falls rapidly behind, so that when hydrogen has come to the 200,000-km level, calcium has passed the 300,000-km level (a common height for eruptive prominences), and the disparity continues to increase.

This considerable difference should be easily observable if it actually exists. It can be expressed as an average velocity difference whereby there would be a tendency for the calcium to blow through the hydrogen at a rate of 100 km/sec at the 100,000-km level. W. H. McCrea[16] has considered the resistance of one gas moving through another where the conditions are like those in the prominences and concludes that no large separation can actually take place.

These results for the Milne effect mean that, if we admit that hydrogen and calcium are balanced by light-pressure against gravity, the consequences are (1) small upward velocities of about 10 km/sec, prevalent in quiescent prominences and the chromosphere, will in a few minutes increase to several hundred kilometers per second, in both hydrogen and calcium, from which it is hard to see how the chromosphere can long endure; (2) there will be a tendency to a large separation of hydrogen and calcium; (3) the time-height diagram will be a continuous curve; (4) the velocities will be higher than any ordinarily observed; (5) a reverse Milne effect should take place on streamers and knots returning to the chromosphere, thus offering a resistance which would also separate hydrogen and calcium. Probably we do not know how to compute light-pressure from the spectrum, for it can hardly be of the magnitude sometimes assumed. Without a balanced gravitational field, on the other hand, we cannot explain the laws of motion of eruptive prominences. The indications are that electrical fields must be considered.

PROMINENCES OF CLASS III

A description of the activity that occurs in the chromosphere and the coronal space above a sunspot-group must come from a study of many individual prominences. During the last two years about

[16] *Ibid.*, p. 509.

thirty cases have been observed at the limb and six on the disk with the motion-picture equipment at Lake Angelus.

Seen on the limb, the chromosphere above a spot-group is in a state of continuous disturbance. The principal feature is the surges which rise and fall; there are also minor disturbances which seem to travel along the chromosphere, although this effect is probably one of a number of independent movements. The most significant feature is a raised knob or hill, sometimes hemispherical, which stands in the center of the group; a height and width of about 30,000 km would be representative dimensions. Sometimes this elevation is flat-topped, like a desert mesa. When projected on the screen, small variations in elevation are seen in several parts of its upper surface, and sometimes the whole top will twist or momentarily break apart. These we have called "cap prominences," since they seem to cover the intensely active area over a sunspot-group; their surface brightness is about equal to the photosphere.

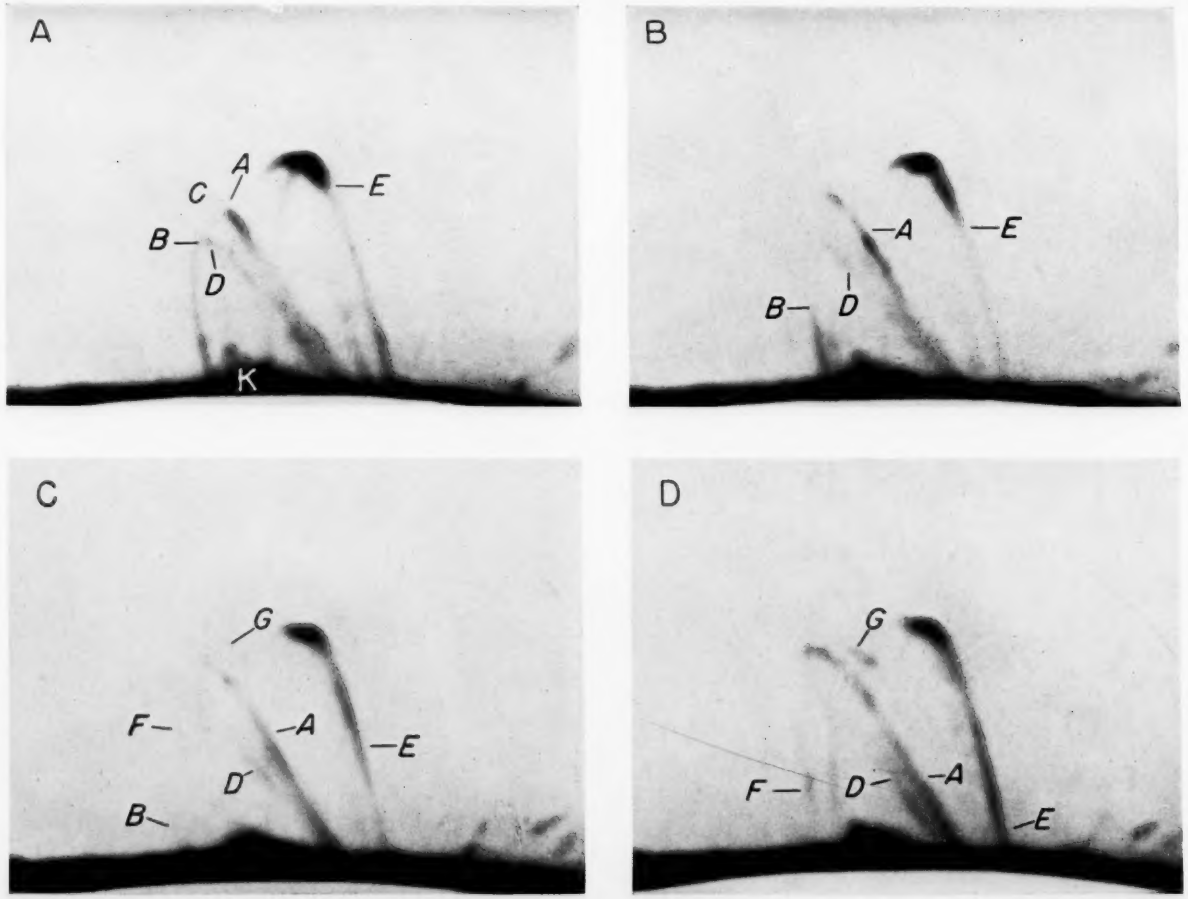
Large sunspot-groups on the limb are sometimes accompanied by two or three of these prominences. They are more enduring than most features of a class III prominence, usually preserving their position, though with changes in shape, over a period of several hours. Occasionally they become eruptive and rise in a spray, then fade back to more or less the original condition.

Plate XII shows both these phenomena. Changes in the upper surface of the cap prominence are distinctly seen taking place within a period of a few minutes.

MOTION IN LOOPS AND ARCHES

In a previous paper[1] we have shown that in a class III*a* prominence matter seems to form above a spot area and pour down into it. Usually a faint point of light appears above the spot area, motionless, brightens for a period of some minutes, then either feeds streamers or bodily moves down into the spot area.

We now have the case where, after the stationary state, the luminous spot lengthens, arches, and then feeds a single streamer from each end to the spot area, thus completing a loop. We have already called attention to the giant arches, 100,000 km or more high, in which matter is seen descending down either branch to the



CLASS III*b* PROMINENCE

Over sunspot-group in P.A. 92°, June 8, 1937. A–D, G.C.T. 12^h44^m, 12^h49^m, 12^h54^m, 12^h59^m. Note that the crest *C* of the inclined loop in A is broken and that the two branches *A* and *B* flow down to the chromosphere. *D* and *E* in the other loop do likewise. Note activity in the cap prominence K.

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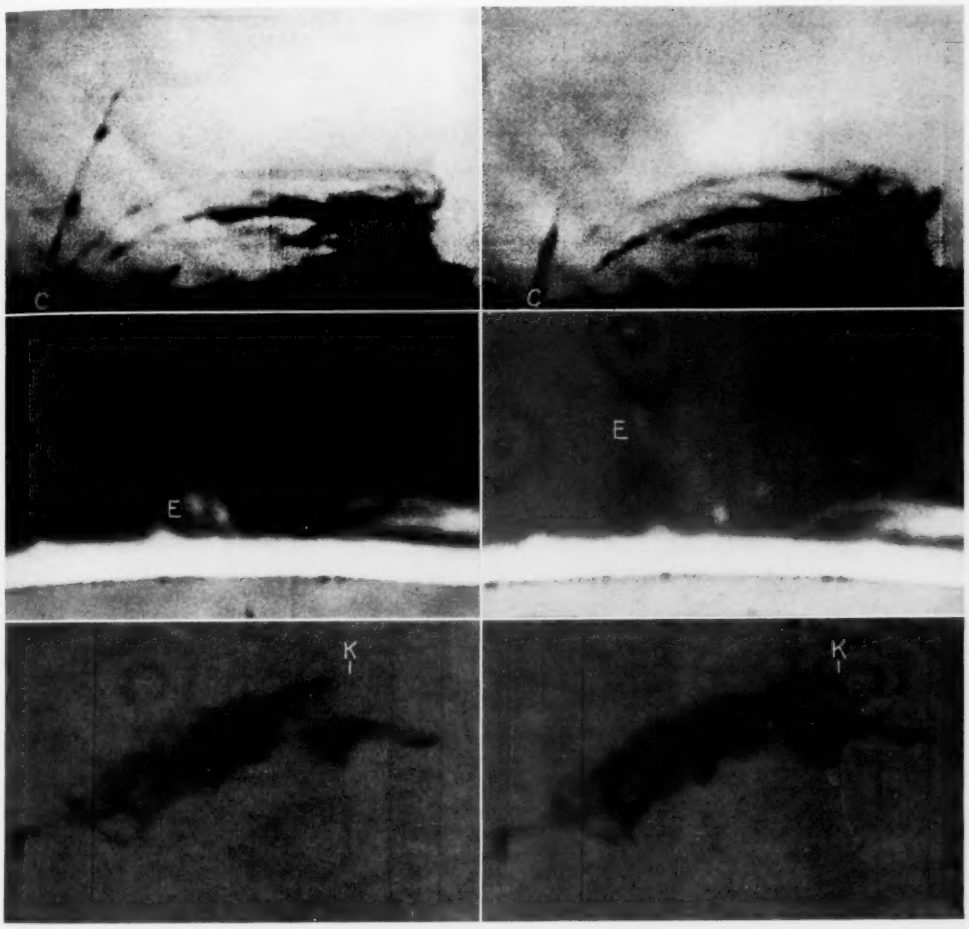
a

b

c

b)
No

PLATE XIII



CORONAL PROMINENCE, EJECTION, AND ACTIVE PROMINENCE

a) Coronal prominence entering center of attraction C. October 4, 1936. G.C.T. 16^h21^m, 16^h32^m. *b*) Ejection E of July 29, 1937. G.C.T. 16^h07^m, 16^h10^m. *c*) Active prominence on the disk July 6, 1937. Note absence of any marking at center of attraction K. G.C.T. 15^h14^m, 15^h22^m.

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chromosphere. It now appears that the same action takes place in the loops of the fountain, class III*b* prominences.

Plate XII shows four stages of such a prominence. Here all the motions are downward to the spot area marked by the cap prominence K, and the inclined arch breaks at the top *C*, the two branches *A* and *B* moving into the chromosphere in succeeding exposures.

Even when only one branch of the arch can be seen, downward motion only is visible. A class III prominence projected on the screen gives the impression that all motion is toward the spot area. There are, however, occasional surges and, more rarely, the expulsion of knots of material more or less round, which leave the chromosphere and do not return. These we have called "ejections" (Pl. XIII*b*).

Figures 2, 3, and 4 show time-distance diagrams of measures of velocities of knots and streamer-ends in prominences of the sunspot type. In general, these obey the first law of eruptive prominences, but not usually the second. Table 5 lists those which approximate the second law within reasonable limits. As in Table 1, V_0 indicates the observed velocities and F their ratios. Of forty-seven plots of motions in class III prominences which show changes in velocity, nine therefore obey the second law and two show distinct curvature. In addition, seven plots show no change in velocity. The strongly curved diagrams, Nos. 6 and 7 of Figure 2, will be discussed under "Projection Factor."

The highest velocity shown in Table 5, which is reached at the chromosphere, is 213 km/sec for June 24. Velocities of 100 km/sec are quite common, and 150 km/sec occur in several cases. There is a tendency for the streamers and knots to show approximately the same velocities where they enter the chromosphere from any given prominence. The best example is the class III*b* prominence of July 29, with velocities at the chromosphere of 117, 78, 82, 84, 78, and 86 km/sec.

Class III prominences are subject to both the magnetic field of the spot, which can be measured, and an electrical field at the same point, which we assume is the cause of its motion. The ionized atoms of the prominence in motion are essentially an electric current, which produces a magnetic field at right angles to the direction

of motion; hence, there should be a force component between this field and that of the spot which would direct the motion in a spiral down the magnetic lines of force.

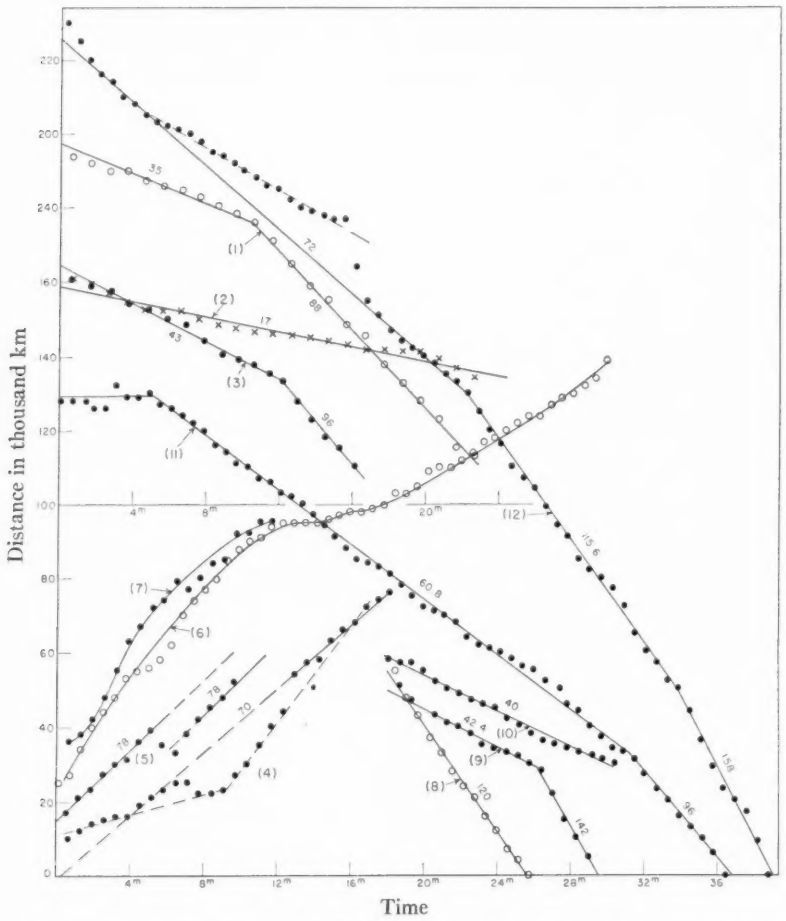

FIG. 2.—Motions of streamers and knots in class III*a* prominences: (1) to (3), June 15; (4) to (12), July 19, 1937.

We have seen little evidence of this spiraling in class III prominences—in the great bulk, none. This must be explained by the circumstances that there may be a nearly equal number of electrons moving in the same direction as the radiating atoms and that the velocities are so high that the component produced by the magnetic

field of the spot acting on the small field produced by the negligible proportion of isolated ionized atoms in the prominence is too small

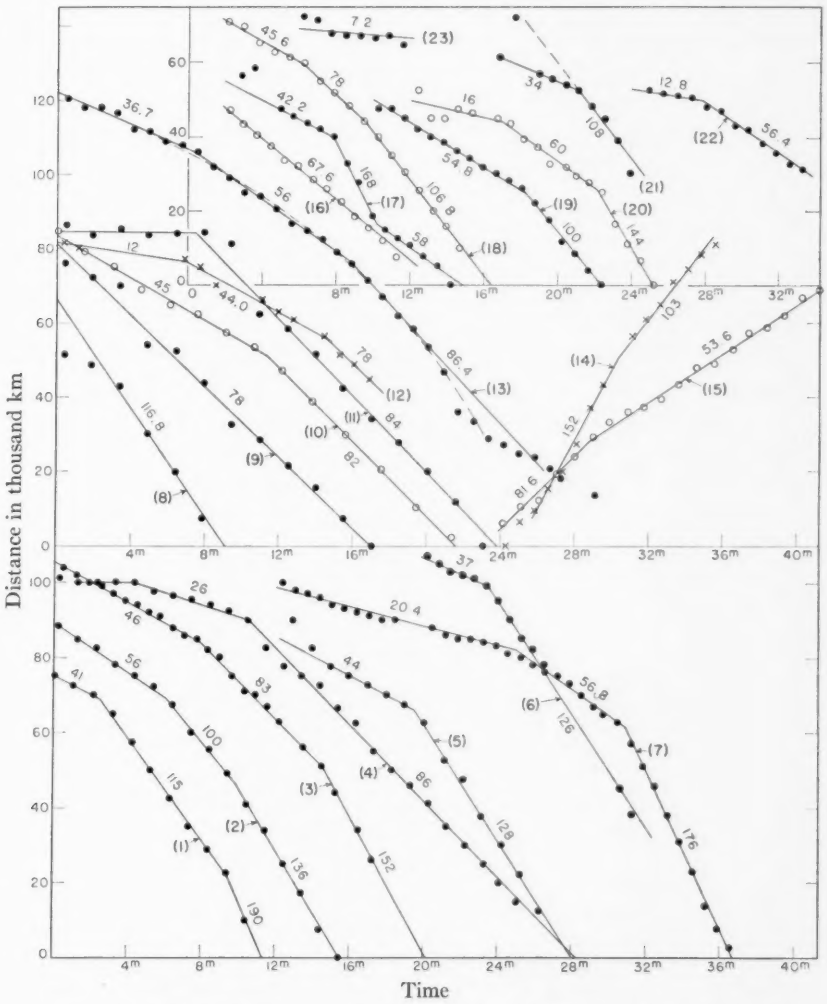

FIG. 3.—Motions of streamers and knots in class III*b* prominences: (1) to (7), June 8; (8) to (15), July 29; (16) to (23), September 17, 1937. Nos. (14) and (15) refer to the ejections.

to produce an observable displacement. On the other hand, we would expect that the electronic content would act almost independently of the ionized atoms.

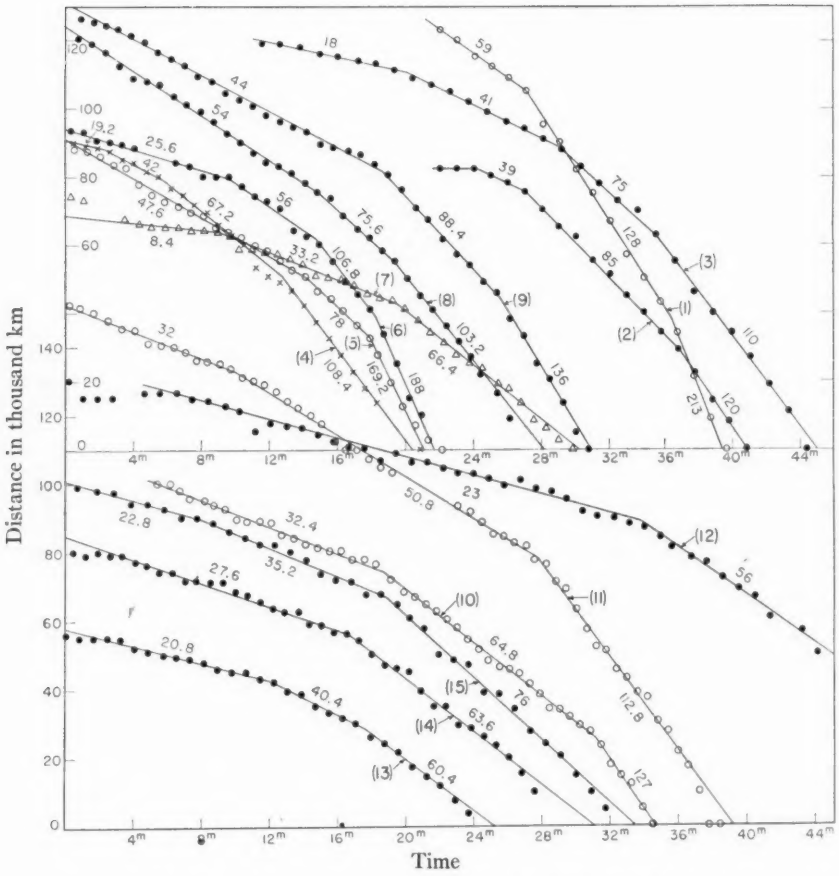

FIG. 4.—Motions of streamers and knots in class III*c* prominences: (1) to (3), June 24; (4) to (9), July 28; (10) to (11), August 1; (12) to (15), August 15, 1937.

TABLE 5

Date	Class	V_0	F
1937, June 8....	III*b*	46, 84, 152	2, 2
		44, 128	3
		20.4, 56.8, 176	3, 3
24....	III*c*	59, 128, 213	2, 2
July 28....	III*c*	25.6, 56.0, 106.8, 188.0	2, 2, 2
		8.4, 33.2, 66.4	4, 2
29....	III*b*	45.0, 82.0	2
Aug. 1....	III	32.4, 64.8, 128.0	2, 2
Sept. 17 ...	III*b*	54.8, 100	2

SURGES AND EJECTIONS

The surges,[1] which are emitted from sunspot areas at almost any angle to a radius, have the appearance of continuous sheets of chromospheric matter which move out and are sucked back again nearly along the same path into the chromosphere without breaking continuity. The smaller surges may appear as bumps in the chromosphere or rise 20,000–50,000 km in the form of jets or spikes and sink back. Their duration is only 10 or 20 minutes and, in an active spot-group, they may reappear at nearly the same point at irregular intervals of about 20 minutes. One of these was observed on July 21, 1937, over the great sunspot-group which appeared for the first time at the east limb in latitude N. 31° on the following day. This surge rose and fell at intervals of about 20 minutes throughout the day. Figure 5, No. 1, shows the time-height diagram over about 4 hours. The general characteristics of these curves are: a rapid, nearly uniform rise; a fluctuation at the crest; and a less rapid, nearly uniform subsidence.

A giant surge appeared over the same spot-group on August 1 on the west limb, and Nos. 2–5 of Figure 5 show the time-height plots. The time scale has been expanded to show the linear character of the rise and fall. The velocities for the large and small surges are about the same; hence, the duration is about proportional to their rise. The motion in the small surges was nearly vertical, while the large ones were projected at an angle of 60° to the vertical.

Whether any considerable chromospheric matter actually leaves the sun with a surge is still an open question. The smaller examples usually show a faint extension of the head, which may be half as long as the prominence itself near the time of its greatest rise. The giant surges sometimes break into a spray, as if lateral expansion had taken place; and at other times, at the highest rise or extension, a small round cloud, called a *secondary*, will leave the end of the prominence; but in one case, on August 1, 1937, this seemed to return. It is possible, however, that the secondary and the return knot were not identical, as the secondary vanished before the knot appeared.

Plate XIII*b* illustrates a rare phenomenon in which small lumps of chromospheric matter, having the appearance of hollow globes or

rings 6000 or 7000 km in diameter, are ejected from a spot area in a nearly straight line, making an angle of 40° to the radius. Five of these ejections were observed to leave the spot-group Mt. W. 5489

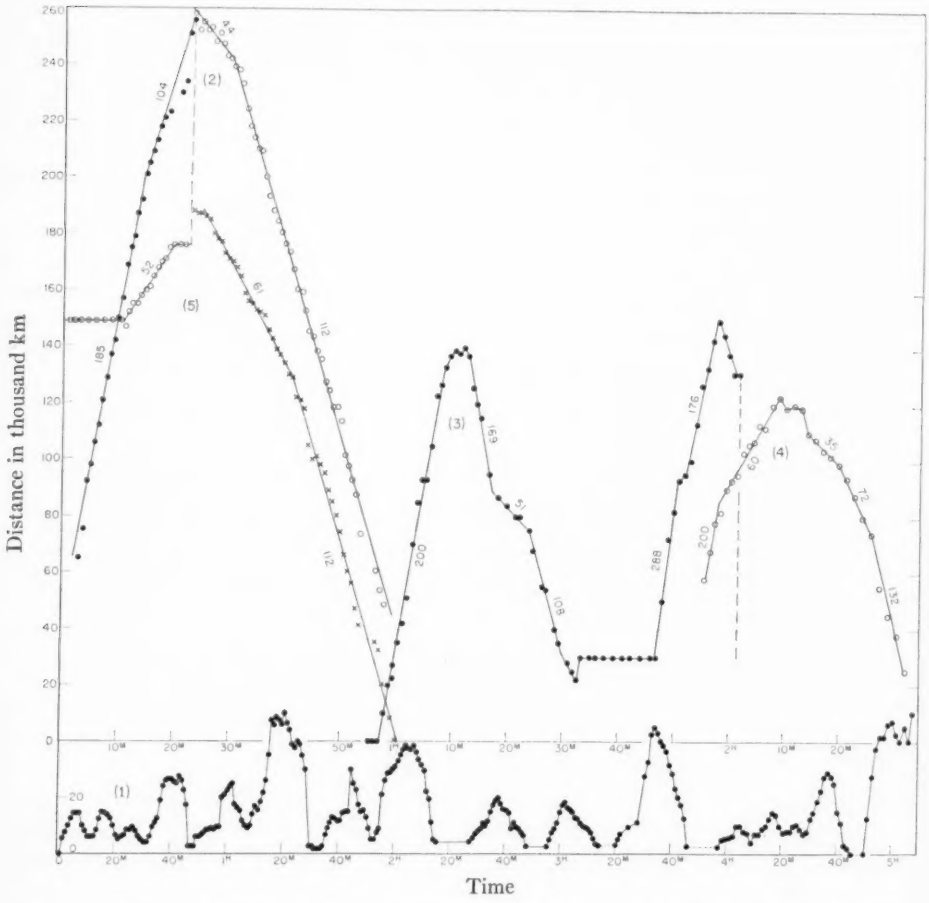

FIG. 5.—Motions of surges (class III*d*): (1), July 21; (2) to (5), August 1, 1937. Broken line in (2) due to change of origin as prominence returned along slightly different path.

in latitude N. 25° on the east limb, July 29, 1937. July 29 and 30 were cloudy at Mount Wilson, but on the two following days bright chromospheric eruptions of intensities 3 and 1 were observed in this position.

The intensities of these objects are usually rather low, those illustrated being about 0.1 that of the chromosphere at the limb. A number have been seen which are near the limit of vision. The time-height diagrams of both measurable cases, Nos. 14 and 15 of Figure 3, show a decrease in velocity, in contrast to the increases found for other types of prominences. It is possible that this singularity is not general with these objects. When the film is projected as a motion picture with high illumination, the ejections have strongly the appearance of smoke-rings. Two ejections, as bright as ordinary prominences and elongated about four times their width, were observed on April 29, 1937, over the spot-group Mt. W. 5342. They were too near the edge of the frame to be fully observed.

CENTERS OF ATTRACTION AND THE METAMORPHOSIS OF QUIESCENT PROMINENCES

A quiescent prominence 55,000 km high and about 12,000 km thick came over the east limb of the sun on July 1, 1937, in latitude S. 42° and extended over nearly 90° in a parallel of latitude. This was followed with the 30-foot collimator in the spectroheliograph, set for *Hα*, during the following week, as it crossed the disk as an absorption marking and passed into the active stage by development of centers of attraction which finally pulled it back into the chromosphere. Even in the quiescent state, the rootlike connections with the chromosphere can be seen pouring into it when the film is projected as a motion picture. As final dissolution takes place, the streamers become more pronounced, as shown in Plate XIII*c*, pouring into the center of attraction K. There is no marking on the chromosphere at K; hence, these centers of attraction are not floccular areas or dark markings of any kind. We therefore suppose that the center is located some distance below the chromosphere.

One of the outstanding differences between centers of attraction and sunspots is in the number and distribution of separate streamers that enter them. Over sunspots observed at the limb a cluster of streamers is usually seen entering at various inclinations and on both sides of the spot. In addition to these isolated streamers, which seem to form spontaneously in the space above, prominences on either side frequently pour into the spot area. On the other hand,

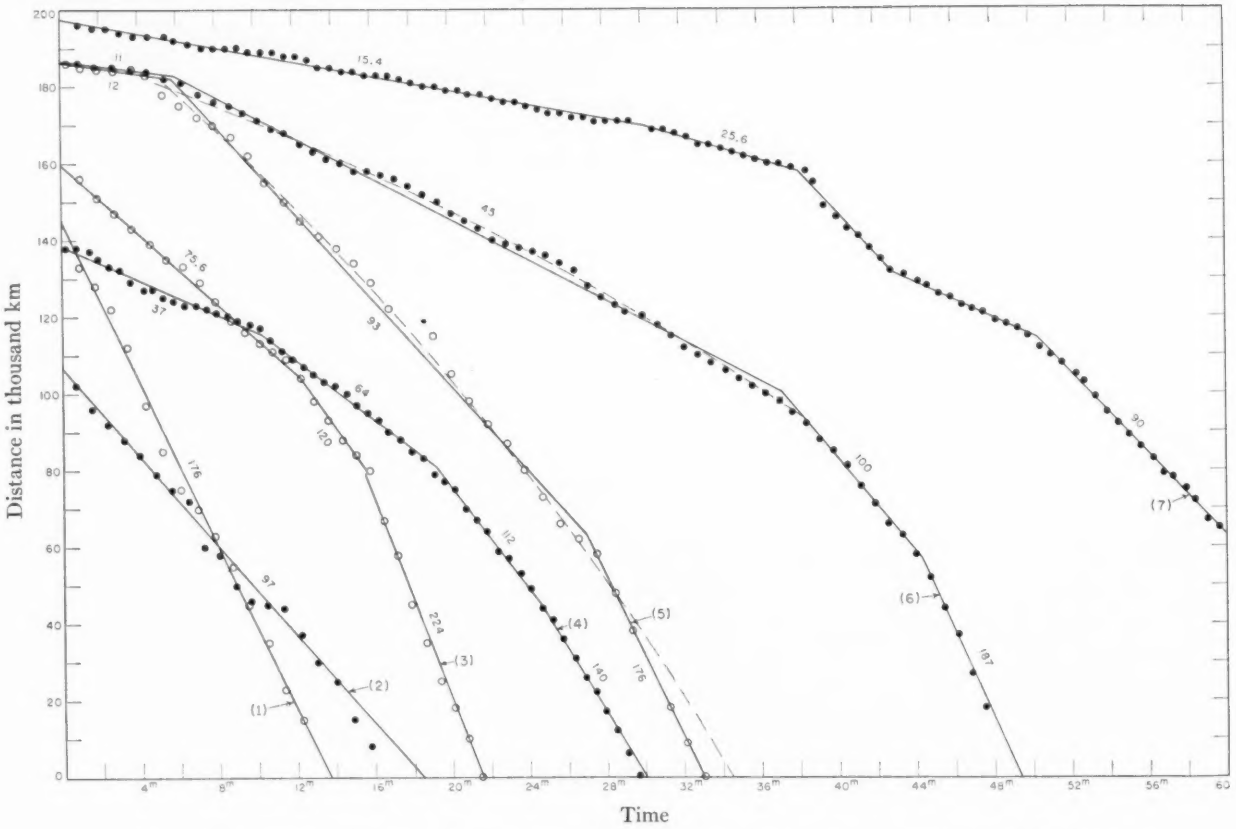

FIG. 6.—Motions of knots and streamers into center of attraction in the class I prominence of June 11, 1937

centers of attraction almost invariably pull streamers from a prominence on one side only. No streamers form in the space above and, in addition, only an occasional coronal streamer enters from outside space at high speed.

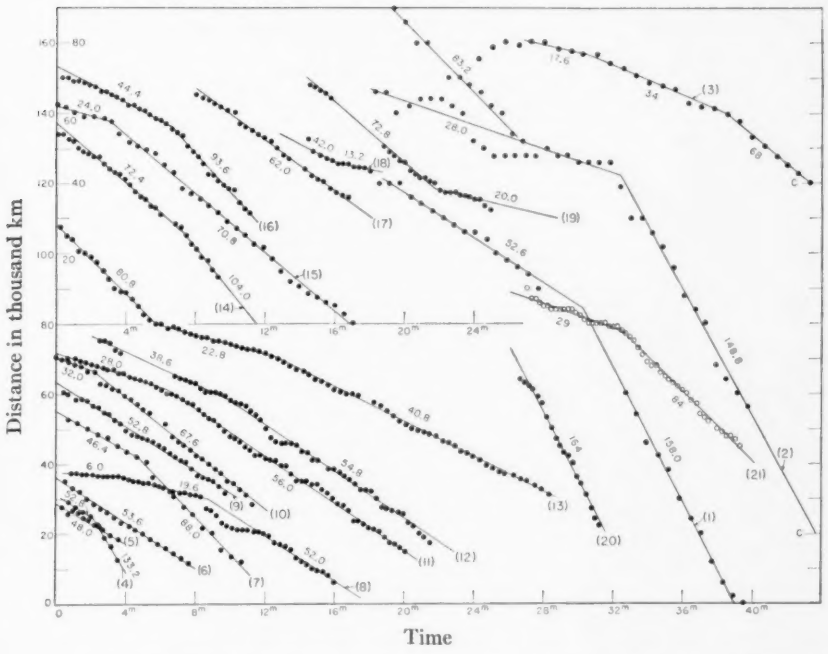

FIG. 7.—Motion of knots and streamers in class I prominences: (3), July 15; (4) to (19), September 24; (20) to (21), September 26, 1937. Nos. (1) and (2) refer to the coronal prominence of October 4, 1936 (Pl. XIII).

We have only one exception, that of the active prominence of May 11, 1937, which appeared on the east limb near the solar equator, pouring into a center of attraction located S. 6°. A cluster of arches formed now and again in latitude S. 11° and poured into the center within 10,000 km of the point where the streamers from the active prominence entered the chromosphere. If the trajectories of the knots and the streamers are extended to intersect, they meet at a point 10,000 km below the chromosphere, possibly a little below the photosphere. This may be an indication of the level of the center of force. If we do the same thing with a sunspot-type prominence, we

find knots and streamers entering in a nearly perpendicular direction over a considerable area, sometimes 80,000 km in diameter, as in the class III*a* prominence of April 29, 1937, in latitude N. 25° on the west limb over spot-group Mt. W. 5342. The existence of many closely packed force centers would explain this. Nevertheless, if we outline the contour of the prominence, including the streamers that

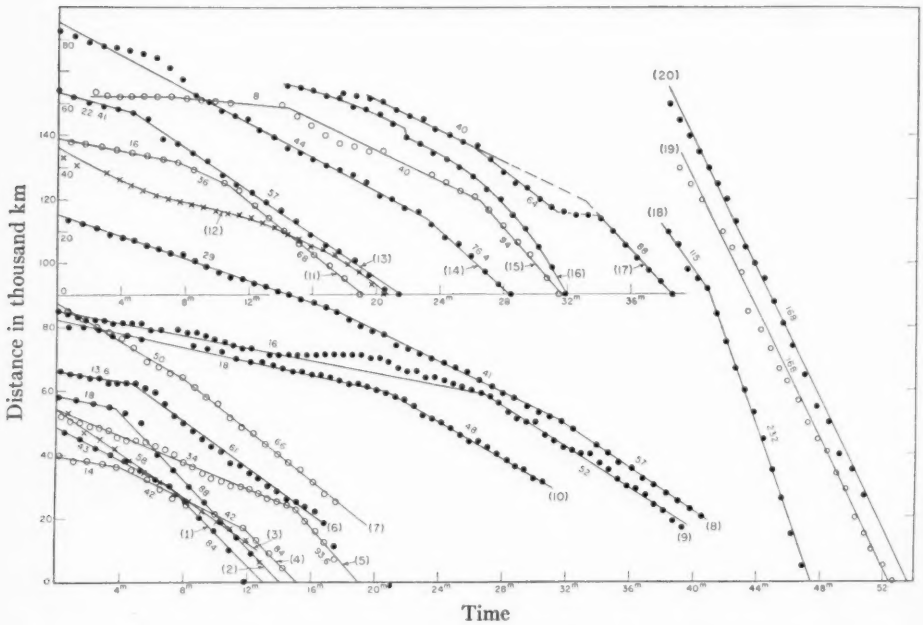

FIG. 8.—Motions of knots and streamers into centers of attraction in the class I prominences: (1) to (6), May 11; (7) to (8), May 17; (9) to (10), June 27; (11) to (17), May 29. Nos. (18), (19), and (20) refer to coronal prominences of June 6, 1937.

enter the chromosphere at a high slant, it converges to a point indicating a depth of 40,000 km for the force center. One of the most closely packed III*b* prominences, photographed at Mount Wilson on December 9, 1929, gave, by contour extension, a center depth of 20,000 km.

The velocities of twenty-seven of the knots and streamer-ends in class I prominences are shown in Figures 6, 7, and 8. Except Nos. 12 and 16 in Figure 8 and a possible slight curvature in Nos. 5 and 6 of Figure 6, all obey the first law of motion of eruptive promi-

nences. Those which obey the second law are listed in Table 6. Here, ten out of twenty that show changes in velocity follow the second law approximately, a proportion of 50 per cent. The entire group of 1 to 6 in Figure 8 of the prominence of May 11 follow the second law.

We have already called attention to the scarcity of streamers in the *Hα* spectroheliograms of prominences when compared with those taken in Ca^+. This now appears to be due to the use of narrow slits and considerable underexposure for these features of a prominence. The class I prominence of September 24 and the class II prominence

TABLE 6

	No.	V_0	F
Fig. 8	1	43, 84	2
	2	18, 88	5
	4	14, 42, 84	3, 2
	5	34, 93.6	3
	9	16, 52	3
	11	16, 36, 68	2, 2
	15	8, 40, 94	5, 2
	18	115, 232	2
Fig. 6	5	12, 93, 176	8, 2
	6	11, 45, 100, 187	4, 2, 2

of September 26, 1937, were photographed in *Hα* with slits 0.4 mm wide, instead of the customary 0.15-mm slits. The resulting pictures are very much like those taken in Ca^+. The plots in Figure 7 abundantly show that the laws of prominence motion apply in *Hα*, as well as in Ca^+.

PROJECTION FACTOR AND DOPPLER DISPLACEMENT

Numbers 6 and 7 of Figure 2 refer to arches observed July 19, 1937, over a sunspot, the first forming near the chromosphere, rising and returning nearly in the plane of the observer. Almost simultaneously a higher arch in a somewhat different plane exhibited the same phenomenon. We suppose the curvature is largely due to the observer's being in the plane of motion, which is really uniform; the inflex in No. 6 is at the crest of the trajectory, as this explanation would require. The effect observed here, according to a discussion

by H. Sawyer and J. Brodie,[17] is about the maximum that may be expected for a streamer moving with uniform motion along a trajectory in the plane of the observer.

Doppler displacement sufficient to affect the exposures has been observed on two occasions. A class III*b* prominence observed on July 19, 1937, just previous to and over the same spot-group as the arches referred to above, expanded with a velocity of 240 km/sec as measured across the line of sight and parallel to the chromosphere. It is reasonable to suppose that the line-of-sight velocity was of the same order, which would move the line out of our slit, then limited to velocities within the range ± 114 km/sec. This would account for the broken appearance. The effects of Doppler displacement will be a subject for investigation in the approaching observing season.

CORONAL PROMINENCES

Plate XIII*a* shows two exposures on an active prominence in latitude S. 15° on the west limb of the sun on October 4, 1936, pouring into a center of attraction in latitude S. 29°. The streamers are entering the center C with velocities of 75–100 km/sec along curved trajectories. At C, where the topmost streamer enters, will be seen another knotted streamer nearly vertical but inclined toward the prominence moving into C. Its origin seems to be 160,000 km above C. A second streamer also entered C as the first streamer disappeared into it. Plots 1 and 2, Figure 7, show the time-distance diagrams of these streamers. Both reach a velocity of about 150 km/sec, and both obey the two laws of motion of eruptive prominences.

That these streamers appeared at great heights above the chromosphere and were not connected with it or with other prominences outside the frame of the motion-picture camera is verified by three exposures taken at Mount Wilson with the 13-foot spectroheliograph at 16^h20^m, 23^m, and 25^m, respectively, within the interval covered by the exposures at Lake Angelus shown in Plate XIII. We therefore conclude that their origin is in the coronal region, and we have called them "coronal prominences." Numbers 18, 19, and 20 in Figure 8 are the time-distance plots of coronals which appeared with the

[17] *Pub. Obs. U. Michigan*, **7**, 79, 1938.

active prominence of June 6, 1937. The velocities of these coronal prominences are given in Table 7. In the three cases showing more than one velocity, the second, as well as the first, law of motion of eruptive prominences is fulfilled with an accuracy nearly equal to that for the eruptive prominences themselves (cf. Table 1).

All the twelve coronals we have so far seen enter centers of attraction connected with active prominences (located in the sunspot zones) with high velocity (Table 7). We have here the evidence that,

TABLE 7

Date	No.	Fig.	V_0		F
1936, Oct. 4.....	1	7	52.6	158	3
	2	7	28.0	148.8	5
1937, June 6.....	18	8	115	232	2
	19	8	.....	168	.....
	20	8	.....	168	.....

as with sunspot types, prominence material appears high above the active area and pours into it. One of the salient features of these coronal prominences is that they rapidly brighten as they approach the chromosphere. Streamers from ordinary prominences, on the other hand, usually diminish in brightness, owing in part to their increased length, as they approach the chromosphere.

We are indebted to the observing staff at Lake Angelus for much assistance in securing the data used here, and to Mrs. Pettit for a large amount of computing and for the translation of a number of papers needed in the investigation.

McMath-Hulbert Observatory of the University of Michigan,
Carnegie Institution of Washington
Mount Wilson Observatory
May 1938

LINES OF IONIZED BARIUM IN STELLAR SPECTRA*

CORA G. BURWELL

ABSTRACT

Five A-type stars have been found in which the red triplet of *Ba* II is abnormally intense. All data agree that these stars are somewhat fainter than the average of the main sequence, although in later types *Ba* II is strong in giant stars.

This behavior of *Ba* II may be similar to that of *Sr* II in the Fo strontium "dwarfs." A brief comparison of the behavior of *Ca*, *Sr*, and *Ba* is given.

Lines of ionized barium appear in the normal sequence in early A-type stars, their intensity increasing gradually with advancing type. With few exceptions, λ 4554, the ultimate line of *Ba* II, was not seen in the spectra of numerous stars of types earlier than A2, which have been thoroughly investigated.[1] Ionized barium was found to be very strong in α Persei, F5.[2] It is present in the sun and strengthened in spots.[3] In N-type stars λ 4554 is strong and is a distinctive feature of class S.[4]

Although most of the published data refer to λ 4554, the triplet in the red has also been observed. Line λ 6142 was measured in α Canis Majoris, an early A-type star, and in ε Ursae Majoris, Aop.[5] Lines λλ 5854, 6142, and 6497 are conspicuous in α Persei.[2] They were also measured in Nova Herculis 1934;[6] two weak components appeared suddenly the day following maximum light of the nova and behaved much like the two components of neutral sodium.

An investigation of interstellar lines in the visual region at Mount

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 598.

[1] Struve, *Ap. J.*, **74**, 225, 1931; Marshall, *Pub. Obs. Univ. Mich.*, **5**, 137, 1934; Marshall, *Ap. J.*, **82**, 97, 1935; Morgan, *Pub. Yerkes Obs.*, **7**, 133, 1934; Menzel, *Harvard Circ.*, No. 258, 1924.

[2] Dunham, *Princeton Univ. Obs. Contr.*, No. 9, 1929.

[3] C. E. St. John *et al.*, *Revision of Rowland's Preliminary Table of Solar Spectrum Wave-Lengths, Carnegie Institution of Washington Pub.*, No. 396, 1928.

[4] Merrill, *Mt. W. Contr.*, No. 306; *Ap. J.*, **63**, 13, 1926.

[5] Marshall, *Ap. J.*, **82**, 97, 1935.

[6] Merrill, *Mt. W. Contr.*, No. 530; *Ap. J.*, **82**, 413, 1935.

Wilson has given an opportunity to study the behavior of the *Ba* II triplet in numerous A-type stars.

The most favorable line for observation, λ 6142, lies in a well-exposed part of the spectrum.[7] The region of λ 6497 is often underexposed with the emulsion used; but when seen, the line follows the behavior of λ 6142. Line λ 5854 is also present but is the weakest member of the triplet in both laboratory and star.

TABLE 1

A-TYPE STARS HAVING THE RED TRIPLET OF *Ba* II ABNORMALLY INTENSE

STAR	MAG.	TYPE		DATE OF OBSERVATION	REMARK IN HD
		HD	Rev.		
HD 20210.....	6.4	A2	A7	1933, Nov. 6	The lines are narrow. λλ 4077.9, 4128.1, and 4131.1 are well marked
23193.....	5.6	A2	A4	1933, Dec. 6	The lines are narrow and the intensities resemble those in the spectrum of α Cygni
78209.....	4.5	A3p	A7	1937, Dec. 17	f Ursae Majoris. The spectrum is peculiar and probably composite. The hydrogen lines are as intense as in class A3, while the metallic lines resemble those of class F5
172546....	6.1	A2	A7	1933, Sept. 9	
173648 ...	4.3	A3	A7	1928, July 24	ζ¹ Lyrae

The red triplet makes its appearance at about A5 in the stellar sequence, and, like the ultimate line λ 4554, tends to increase slightly in intensity with advancing type. The three lines are present in the sun and strengthened in spots. In later-type stars, banded structure makes it difficult to follow their behavior.

The *Ba* II lines in the visual region were found to be abnormally strong in several A-type stars included in the Mount Wilson program on interstellar lines. The revised types of these stars, listed in Table 1, are based on the strength and general appearance of the metallic lines, chiefly those of *Fe* I.

[7] Its intensity cannot be appreciably affected by a faint iron line at the same wave length.

The only star in which Morgan[1] found λ 4554 to be "well-marked" was HD 78209 (15 f Ursae Majoris). The strength of the red triplet photographed at Mount Wilson shows that this star belongs to the barium group, and it is therefore included in the present study.

The spectra of the five stars in Table 1 are similar enough to warrant treating them as a group. The most conspicuous lines between λ 5850 and λ 6500 are: D2, D1, *Na* I; λ 6142 and λ 6497, *Ba* II; λ 6347 and λ 6371, *Si* II; λ 6138 and λ 6192, *Fe* I; λ 6248 and λ 6456, *Fe* II. The lines of *Ba* II and *Si* II appear to be about equal in intensity. Numerous fainter lines have been identified with *Fe* I and *Fe* II. The region of *Ha* is underexposed except in f Ursae Majoris and ζ^1 Lyrae, in both of which the line has a strong core with broad wings.

ABSOLUTE MAGNITUDE

Absolute magnitudes of three stars obtained from trigonometric parallaxes are given in Table 2. They are in reasonable agreement

TABLE 2

ABSOLUTE-MAGNITUDE DATA

Star	*l*	*b*	*m*	*M*		Equiv. Width		
							Si II	
				Tr.	Sp.*	0.5 (D2+D1)	λ 6347	λ 6371
HD 20210	122°	−18°	6.4	.....	.....	0.31 A	0.21 A	0.13 A
23193	126	−13	5.6	1.8	.....	.20	.....	.19
78209	134	+43	4.5	2.3	2.2	.....	.....	.....
172546	338	−10	6.1	.....	2.7	0.25	0.13	0.12
173648	34	+16	4.3	1.5	1.8	.....	.....	.....

* Spectroscopic absolute magnitudes from *Mt. W. Contr.*, No. 511; *Ap. J.*, **81**, 187, 1935.

with the spectroscopic magnitudes derived from ratios of line intensities which yield correct parallaxes for normal stars.[8]

Some conclusions on absolute magnitude can also be drawn from observations of the sodium lines. The correlation between distance

[8] Adams *et al.*, *Mt. W. Contr.*, No. 511; *Ap. J.*, **81**, 187, 1935.

and intensity of interstellar sodium lines makes it possible to obtain approximate values of magnitude for individual stars.[9]

The sodium lines in Table 2 are little, if any, more intense than would be expected of stellar lines. In HD 20210 and HD 23193, the equivalent widths show rather definitely that interstellar sodium contributes relatively little to the lines. Interstellar absorption of 0.1 A for HD 20210 and 0.07 A for HD 23193 would give distances of 150 and 100 parsecs, respectively, and corresponding magnitudes of +0.5 and +0.6. Stellar origin is not so certain in HD 172546; if interstellar sodium contributes 0.1 A, the absolute magnitude is +0.2. Since the amount of interstellar sodium is probably even less than these assumed values, the stars are probably fainter.

Although the distance-intensity relation should be used with caution for stars of high galactic latitude, it probably gives magnitudes of the right order for the stars in Table 2.

The predominance of stellar sodium in HD 20210 and HD 23193 is borne out by the displacements, which agree with those of other stellar lines. No conclusions can be drawn from the velocities in the case of HD 78209 or HD 172546. The displacement of the sodium lines in HD 173648 lies between that of the stellar lines and that to be expected from interstellar lines. The apparent magnitude of the star, however, makes it doubtful that this is a distant object.

The unindentified interstellar lines, λ 5780 and λ 6284, are extremely weak if present in these "barium stars."

The moderate intensities of the *Si* II lines λ 6347 and λ 6371 in Table 2 appear normal. These lines reach a maximum near Ao and, except in c stars, disappear at about Fo. In c stars, however, the lines are stronger throughout and the maximum is broader; they are conspicuous in ρ Cassiopeiae, cGo.

All data agree in indicating that the stars in Table 2 are not highly luminous; they probably tend to be among the fainter stars of similar types.

The absolute-magnitude effect of *Ba* II is not definite in early A-type stars. Both λ 4554 and λ 6142[5] are present in α Canis Majoris and ε Ursae Majoris, are weak or absent in α Lyrae, and are not seen in the supergiant α Cygni. All observers agree that α Canis

[9] Merrill, *Mt. W. Contr.*, No. 569; *Ap. J.*, **86**, 28, 1937.

Majoris is fainter than the average star of its type; hence the probability that *Ba* II is stronger in the fainter stars of types earlier than A3. Data are lacking for a thorough investigation.

Comparison of spectra of stars of known absolute magnitudes shows that λ 4554 is more intense in the high-luminosity G-type stars ρ Cassiopeiae and δ Canis Majoris than in any other types except N and S.

Menzel[9] compared ε Indi, dK5, with α Tauri, gK5, and found λ 4554 "much stronger" in α Tauri.[10]

In the S-type variables, λ 4554 is more intense near maximum.[11]

The red triplet of *Ba* II is much stronger in ρ Puppis, cF5, than in the low-luminosity star, ξ Pegasi, F3. It is conspicuous in α Scorpii, gM1, and barely seen in 61 Cygni A, dK6.

These comparisons indicate that *Ba* II is more intense in stars of *high* luminosity in types F and later, suggesting the possibility of a change in absolute-magnitude behavior at F. Spectrograms for a more complete investigation are not available.

INTERCOMPARISON OF *Ca*, *Sr*, AND *Ba*

The ultimate lines of the related elements *Ca* II and *Sr* II are present throughout the general sequence of stellar spectra from A to M, and both have flat maxima in K-type stars.[12] The analogous lines of *Ba* II are, in general, less conspicuous than those of *Ca* II and *Sr* II; the maximum, which has not been determined, is probably also flat.

The lines H and K, the ultimate lines of *Ca* II, are unusually narrow in HD 20210 and HD 23193; they appear normal in HD 173-648, ζ[1] Lyrae. The region is underexposed in the other two stars in Table 1.

Lines λ 4077 and λ 4215, *Sr* II, are relatively somewhat stronger than the other lines used for spectroscopic absolute magnitude in the three stars in Table 2. In HD 20210 and HD 23193, however, the lines are not unusually intense.

[10] Attention has been called to the K-type star, HD 116713, in which λ 4554 is abnormally intense. Spectroscopic absolute-magnitude criteria classify this star as a subgiant.

[11] Merrill, *Mt. W. Contr.*, No. 325; *Ap. J.*, **65**, 23, 1927.

[12] Russell, Dugan, and Stewart, *Astronomy*, p. 863.

Line λ 4554, *Ba* II, may be slightly more intense in these stars in which the red triplet is outstanding, but blends make it difficult to compare.

In general, *Sr* II increases in intensity with advancing type, and more rapidly in giants than in dwarfs. Several A-type stars and a few of type F, of which γ Equulei is an example, have λ 4077 and λ 4215 abnormally intense. Other considerations, however, indicate that they are not giant stars.[13, 14] The Mount Wilson type, cF1, and absolute magnitude, −0.2, of γ Equulei depend largely upon the intensity of λ 4077 and λ 4215.

TABLE 3

ANALOGOUS MULTIPLETS IN *Ca* II, *Sr* II, *Ba* II

E.P.	*Ca* II		*Sr* II		*Ba* II	
0.0	3933.68	$4^2S-4^2P^\circ$	4077.73	$5^2S-5^2P^\circ$	4554.04	$6^2S-6^2P^\circ$
	3968.49	.....	4215.54	.....	4934.09	.....
0.6	.....	.....	.....	.....	5853.69	$5^2D-6^2P^\circ$
	.....	.....	.....	.....	6141.73	.....
	.....	.....	.....	.....	6496.92	.....
1.7	8498.06	$3^2D-4^2P^\circ$	.....	.....	.....	.....
	8542.13	.....	.....	.....	.....	.....
	8662.17	.....	.....	.....	.....	.....
1.8	.....	.....	10036.56	$4^2D-5^2P^\circ$	.....	.....
	.....	.....	10327.24	.....	.....	.....
	.....	.....	10914.82	.....	.....	.....

Table 3 shows the corresponding multiplets of the closely related elements *Ca* II, *Sr* II, and *Ba* II. The infrared triplet of *Ca* II behaves in general like the ultimate lines H and K.[15] The *Sr* II lines lie too far in the red for observation. This investigation suggests the behavior of the analogous *Ba* II triplet.

The ultimate lines λ 4227, *Ca* I, and λ 4607, *Sr* I, are more intense in spots than in the sun, and in stellar spectra increase in intensity with advancing type. The line λ 5535, *Ba* I, on the contrary, is absent from both sun and spots and is weak or absent in stars of types M, N, and S.[4] Its presence in stellar spectra is doubtful.

[13] Payne, *Stellar Atmospheres*, p. 170.

[14] Hynek, *Ap. J.*, **82**, 338, 1935.

[15] Merrill, *Mt. W. Contr.*, No. 486; *Ap. J.*, **79**, 183, 1934.

Although the stars in Table 1 with strong *Ba* II lines are apparently fainter than the average stars of the same type, it is not certain that in general the fainter stars exhibit the barium lines.[16] Similar behavior of *Sr* II is suggested by the group of Fo "dwarfs" in which λ 4077 and λ 4215 are abnormally intense.

I am greatly indebted to Dr. Merrill for suggesting the investigation and for much valuable advice.

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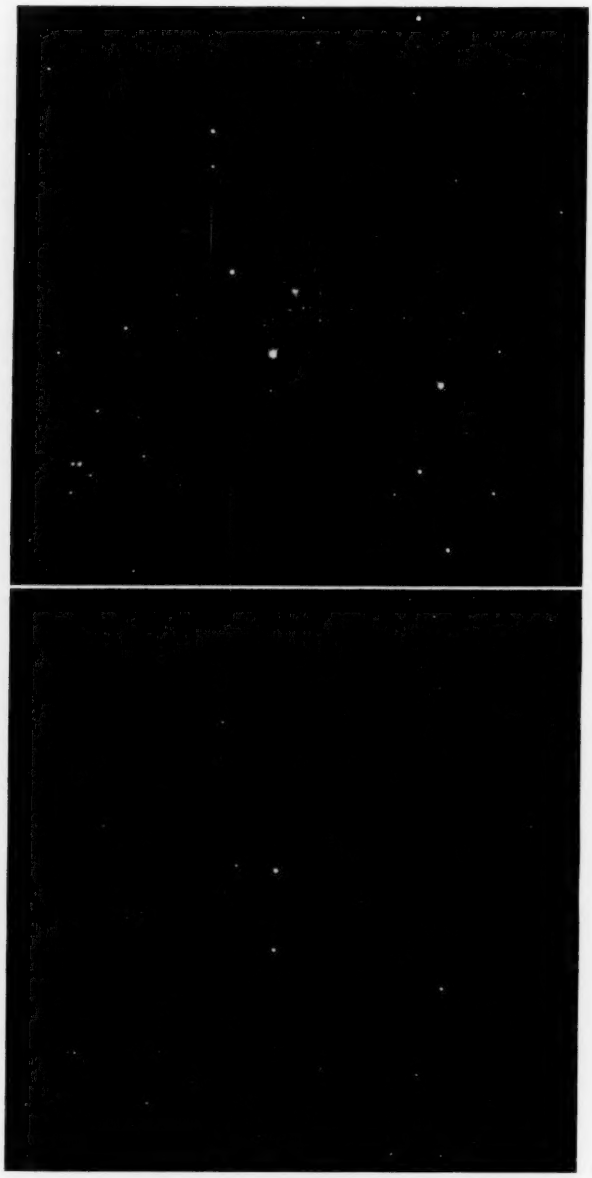
MOUNT WILSON OBSERVATORY
June 1938

[16] λ 4554 is conspicuous in ε Ser (Morgan and Goldena Farnsworth, *Ap. J.*, **76**, 299, 1932) but weak or absent in 73 Dra, although these stars "appear to be physically similar" (Morgan, *ibid.*, **77**, 77, 1933).

PLATE XIV



THE SUPERNOVA IN IC 4182

a) IC 4182 before the appearance of the nova. Exposure 1^h with 60-inch telescope. *b*) The supernova 18 days after its maximum. Exposure 25^m with the 100-inch telescope. Scale, 1 mm = 12″.5.

UNIV. OF MICH.

THE ABSOLUTE PHOTOGRAPHIC MAGNITUDE OF SUPERNOVAE*

W. BAADE

ABSTRACT

A compilation of the photometric data for the 18 supernovae known at the end of 1937 is given. Former estimates have been replaced by photometric magnitudes after a redetermination of the magnitudes of comparison stars on the international system. The mean absolute photographic magnitude of the supernovae, derived from this material, is $\overline{M}_{max} = -14.3 \pm 0.42$ (m.e.) with a dispersion $\sigma_{M_{max}} \approx 1.1$ mag. This result, together with the spectroscopic evidence, fully confirms the view that two classes of novae, common novae and supernovae, exist. Attention is drawn to the curious fact that 72 per cent of the known supernovae appeared in late-type spirals. B Cassiopeiae and the Crab nebula, which may have been galactic supernovae, are discussed.

When, after Ritchey's discovery of a nova in the spiral nebula NGC 6946 in 1917, novae were discovered in rapid succession in other extragalactic systems, a new way had been opened to settle the old question as to the constitution and the distances of these systems. The occurrence of novae in them afforded strong evidence for their stellar constitution. Moreover, their distances could be measured as soon as reliable values for the luminosities of the novae of our own galaxy were available. Nevertheless, the first applications of this method by H. D. Curtis and K. Lundmark were not very satisfactory, because the new data clearly revealed an unexpected feature, a large dispersion in the luminosities of the extragalactic novae.

This feature is well illustrated by the novae of the Andromeda nebula. The first observed in this near-by extragalactic system, the famous nova of 1885, S Andromedae, reached a maximum apparent visual brightness $m_v = 7.2$, a brightness nearly comparable with that of the whole nebula. On the other hand, the many novae discovered in the Andromeda nebula after a systematic search had begun in 1917 were all faint objects, ranging in apparent magnitude from 16 to 18, with a frequency maximum around 17.4. The unusually large range of 11 mag. in the observed luminosities of these

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 600.

novae was disturbing because it indicated either a very large dispersion in the absolute magnitudes or the existence of two groups of novae differing in luminosity by a factor of the order 10,000. A few years later Hubble's investigation of the Andromeda nebula left no doubt that the second alternative had to be adopted. The 85 faint novae observed in this nebula between 1917 and 1927 had all the characteristics of a well-defined group. Their brightness at maximum showed a range of only 3 or 4 mag., with a pronounced frequency maximum at $m = 17.3$. Moreover, their mean absolute magnitude, $M = -5.7$, identified them with the galactic novae.[1]

Compared with these common novae, of which an average of 30 appear annually in the Andromeda nebula, S Andromedae, with an absolute brightness of $M = -15.0$, stands out as a nova *sui generis*. But S Andromedae, though exceptional, is by no means a singular case. For, if we exclude from the list of all novae which have been found in extragalactic systems the few faint novae discovered in other members of the local group (those in Messier 33 and the Magellanic Clouds), which are clearly common novae, the remaining examples are to be classed with S Andromedae because they reached apparent magnitudes comparable with the integrated apparent magnitudes of the systems in which they appeared. From the well-established fact that the integrated luminosities of the extragalactic systems are distributed around the mean value $M = -14.2$, with a rather small dispersion, it follows that the remaining novae reach a maximum luminosity of the order of $M = -14$. The large difference in the absolute magnitudes of these two groups strongly suggests that we are dealing with two different classes of novae.

Data of a quite different nature support this view. Spectroscopic investigation of a number of recent novae of the S Andromedae type has shown that these supernovae exhibit a most peculiar spectrum, different from that of any other known celestial object.[2] It can be

[1] Additional proof that the faint novae of the Andromeda nebula are identical with the novae of our galaxy is afforded by later spectroscopic observations of M. L. Humason, who showed that they exhibit the typical nova spectrum (*Pub. A.S.P.*, **44**, 381, 1932).

[2] For a discussion of the spectra of supernovae see R. Minkowski, *Mt. W. Contr.*, No. 602, soon to appear.

best described as consisting of wide and partly overlapping emission bands, which differ in intensity, shape, and width. This spectrum is faithfully reproduced in all six supernovae for which spectrograms have been obtained so far. Since no single one of the characteristic bands has been identified as to its origin, the spectrum is a mystery at present. And yet, as a spectroscopic definition of a supernova, it serves admirably, for its characteristic features are easily recognized even on spectrograms of very low dispersion. In practice, therefore, we can always decide whether a nova is of the common type or a supernova as soon as its spectrum is available. The totally different character of the spectra of common novae and supernovae puts it beyond doubt that we are dealing with two different classes of objects which, notwithstanding the similarities in their light-curves, probably differ radically in their underlying processes.

Preliminary estimates of the absolute magnitudes of supernovae at maximum have been published by K. Lundmark[3] and by W. Baade and F. Zwicky.[4] A new determination of this constant, based on a revision of the published observational data, is presented in the following pages. In view of the meager and incomplete material now available, the result must be considered as a first approximation. But probably a number of years must elapse before better data will be at our disposal as a result of the systematic search for supernovae which is now under way.

THE LIST OF SUPERNOVAE

If we exclude from the material collected up to the end of 1937 all the faint novae which have been found in the near-by systems M31, M33, and the Magellanic Clouds and which clearly represent common novae, 18 objects (Table 1) are left. Whether all these were supernovae will be discussed later. In making up a preliminary list it is necessary to allow a rather wide margin of uncertainty, since a large part of our present data are due to casual discoveries. Consequently, in most cases the supernovae were probably already well past maximum when discovered. Not included in Table 1 are the

[3] *Lund Medd.*, Ser. 2, No. 74, 1935.

[4] *Mt. W. Comm.*, No. 114; *Proc. Nat. Acad.*, **20**, 254, 1934; *Ann Rept. Mt. W. Obs., 1934*, p. 151.

following objects, which appear in earlier lists of Shapley,[5] Lundmark,[6] and Curtis[7] and which subsequently were found to be either plate defects or ordinary variable stars:

1. *NGC 2403.*—The object announced as a nova by Ritchey (*Pub. A.S.P.*, **29,** 210, 1917) is a plate defect, according to Hubble (*Mt. W. Contr.*, No. 376, p. 25; *Ap. J.*, **69,** 127, 1929).

TABLE 1

LIST OF SUPERNOVAE

NGC	Nova	Year of Maximum	Discoverer
224 (M31)	S Andr	1885	Hartwig
1003	145.1937 Per	1937	Zwicky
2535	100.1901 Cnc	1901	Reinmuth (1923)
2608	3.1920 Cnc	1920	Wolf
2841		1912	Pease (1917)
4157		1937	Zwicky
IC 4182	144.1937 Can. Ven.	1937	Zwicky
4273		1936	Hubble-Moore
4303 (M61)	10.1926 Vir	1926	Wolf-Reinmuth
4321 (M100)		1901	H. D. Curtis (1917)
4321 (M100)		1914	H. D. Curtis (1917)
4424	94.1925 Vir	1895	Wolf (1925)
4486 (M87)	1.1922 Vir	1919	Balanowsky (1922)
4527		1915	H. D. Curtis (1917)
5236 (M83)		1923	Lampland
5253	Z Cent.	1895	Fleming
5457 (M101)	SS U.Ma	1908/1909	Wolf
6946		1917	Ritchey

2. *NGC 3147.*—Mrs. I. Roberts' star near NGC 3147 (*A.N.*, **197,** 57, 1914) is a variable which appears as of magnitude 15.5 on a Mount Wilson plate of March 18, 1923. The star is situated far outside the nebula.

3. *NGC 7831.*—The suspected nova on Plate 67 of *Lick Obs. Pub.*, **8,** 1908 (Mrs. Roberts, *C.R.*, **173,** 1072, 1921), is a plate defect, according to Lundmark, who examined the original plate (*Pub. A.S.P.*, **34,** 53, 1922).

[5] *Pub. A.S.P.*, **29,** 214, 1917.

[6] *K. Sv. Vetensk., Handl.*, **60,** No. 8, 1920.

[7] *Handb. d. Astrophys.*, **5,** Part II, 860, 1933.

THE OBSERVATIONS

Since a considerable part of the published magnitudes of supernovae are at best rough estimates, they are of little value in their original form. To obtain reliable photometric data, I have redetermined the magnitudes of the reference stars by polar comparisons whenever the photometric system was in doubt. In a few cases, in which no further information accompanied the original estimates, the discoverers kindly provided either new estimates referred to a set of convenient comparison stars or prints from the original plates which allowed a complete redetermination of the magnitudes. A comparison between the original estimates and the revised magnitudes has shown that this precaution was well justified, since the estimated magnitudes are, on the average, 1.7 mag. brighter than the photometric values. Without it, the mean absolute magnitude of the supernovae derived from the original material would have been a magnitude brighter than that obtained from the revised values. A summary of the final observational data follows. With one exception (S Andromedae) all magnitudes are photographic.

NGC 224 Sb S Andr. $m - M = 22.2$

A light-curve reduced to the Harvard visual system has been published by K. Lundmark (*K. Sv. Vetensk. Handl.*, **60,** No. 8, 1920).

vis $m_{max} = 7.2$.

NGC 1003 Sc 145.1937 Per. $m - M = 26.8$

A light-curve, based on Bergedorf, Palomar, and Mount Wilson observations will be published in *Mt. W. Contr.*, No. 601; *Ap.J.*, **88,** in November issue, 1938. It shows that the supernova was caught about a week before maximum.

pg $m_{max} = 12.8$

NGC 2535 SBc 100.1901 Cnc

The nova occurs on two Heidelberg plates with the following magnitudes:

1901 Jan. 10, 16.4; 1901 Jan. 18, 14.7

To Reinmuth's original magnitudes (*A.N.*, **221,** 47, 1924) has been added the correction indicated in *Harvard Bull.*, No. 804 to reduce them to the international system. Plates taken at Harvard within a month before and after the Heidelberg observations suggest that the nova did not become brighter than pg $m = 14$ (*Harvard Bull.*, No. 804).

NGC 2608 SBc 3.1920 Cnc

Most of the following magnitudes are unpublished data obtained at the Hamburg and the Mount Wilson observatories. Wolf's estimates have been reduced to the international system.

1920	Feb.	8	12.9	Wolf, *A.N.*, **210,** 374, 1920
		15	13.5	Baade, 40-inch Bergedorf
		18	13.6	Baade, 40-inch Bergedorf
	Mar.	10	13.7	Wolf, *Beob. Zirk.*, **2,** No. 14, 1920
		10	13.9	Hubble, 10-inch Cooke
		11	13.4	Baade
		21	13.7	Hubble, 60-inch
	Apr.	8	14.4	Hubble, 10-inch
	Oct.	18	15.7	Baade
		18	15.6	Baade
1921	Apr.	3	17.5	Baade
1922		22	>20	Hubble, 100-inch

A few visual magnitudes (in the Harvard system) are given by K. Graff, *Beob. Zirk.*, **2,** No. 30, 1920:

1920	Feb.	15	12.05
		27	12.21
	Mar.	10	12.31
	Apr.	14	13.28

The photographic and visual light-curves run parallel to each other, indicating a color index of +1.1. The observations belong to the descending branch of the light-curve.

NGC 2841 Sb

The nova appears on one plate only:

1912 Feb. 19 mag. 18 Pease, *Pub. A.S.P.*, **29,** 213, 1917

Pease's own estimated magnitude (16) is too bright and has been replaced by the foregoing value.

NGC 4157 Sc

The few observations (Zwicky, *Pub. A.S.P.*, **49,** 204, 1937) indicate that at the time of the discovery the nova was already well past maximum.

IC 4182 Sc $m - M = 24.8$

The light-curve (*Mt. W. Contr.*, No. 601), based on Palomar and Mount Wilson data and a pre-maximum observation by Leutenegger, indicates that the nova was discovered shortly after the maximum.

$$\text{pg } m_{\max} = 8.2 .$$

NGC 4273 Sc Member of the Virgo Cluster.
$m - M = 26.7$

The light-curve has been published by the writer (*Pub. A.S.P.*, **48,** 226, 1936). The observations are sufficiently well distributed to make it certain that the maximum brightness was close to

$$\text{pg } m_{\max} = 14.4 .$$

It was the first supernova of which good slit spectra were obtained. For a description of spectra fifty and one hundred days after maximum, see M. L. Humason and W. Baade, *Pub. A.S.P.*, **48,** 110 and 226, 1936.

NGC 4303 SBc 10.1926 Vir. Member of the Virgo Cluster. $m - M = 26.7$

New estimates by K. Reinmuth and a determination of the magnitudes of the comparison stars on the international system led to the following revised magnitudes.

1926	Apr.	15	>17
	May	9	14.3
		12	14.7
		17	14.6 (*Harvard Bull.*, No. 836)
	June	1	15.5

Slitless spectrograms were obtained at the Lick Observatory by C. D. Shane (*Pub. A.S.P.*, **38,** 182, 1926) and at Mount Wilson by Duncan and by Nicholson. One of the Mount Wilson spectra is

reproduced in Humason's paper (*loc. cit.*) on the supernova in NGC 4273.

NGC 4321 Sc Member of the Virgo Cluster.
$m - M = 26.7$

Two novae have been found by H. D. Curtis in this nebula (*Lick Obs. Bull.*, **9**, 108, 1917). Film prints of the original plates, kindly provided by the Lick Observatory, made it possible to redetermine the magnitudes of the novae on the international system, as follows:

No. 1:

1901 Mar. 17, 15.6; Apr. 16, 16.2; Apr. 19, 16.2

The nova was probably discovered on the descending branch of the light-curve.

No. 2: This nova appeared on one plate only:

1914 Mar. 2, 15.7

NGC 4424 SBb Pec. 94.1925 Vir. Member of the Virgo Cluster. $m - M = 26.7$

Wolf's estimates (*A.N.*, **226**, 75, 1925) have been reduced with new magnitudes of the comparison stars:

1895 Mar. 17, 12.5; Apr. 14, 12.9; Apr. 15, 12.9

NGC 4486 E0 1.1922 Vir. Member of the Virgo Cluster. $m - M = 26.7$

New estimates by I. Balanowsky and determination of the magnitudes of the comparison stars on the international system led to the following revised magnitudes.

1919	Feb. 24	12.3	Balanowsky
	Mar. 3	12.4	*Harvard Bull.*, No. 803
	22	14.5	Balanowsky
1920	Feb. 26	19.5	Hubble, *Pub. A.S.P.*, **35**, 261, 1923
	Mar. 17	20.0	*Ibid.*

The observations of the first four weeks suggest that the nova was discovered when still near maximum.

NGC 4527 Sc Member of the Virgo Cluster.
$m - M = 26.7$

Curtis' estimates (*Lick Obs. Bull.*, **9,** 108, 1917) should be replaced by the following magnitudes on the international system.

1915 Mar. 20, 15.5; Apr. 16, 16.4

NGC 5236 Sc $m - M = 24.8$

An excellent unpublished light-curve by C. O. Lampland (extending over sixty days) shows that the nova stayed at the constant brightness $m = 14.0$ from May 5 to May 15, 1923, after which it dropped to $m = 15.7$ at the beginning of July. It is difficult to decide at present whether the constant brightness during the first ten days indicates that the nova was near its maximum, or whether it was a halt on the descending branch of the light-curve. If the first assumption is true, the absolute magnitude of the nova at maximum was only -10.8.

NGC 5253 Irr. Z Centauri

Photographic magnitudes and a light-curve of the nova are given in *Harvard Ann.*, **84,** No. 7, 1923. The maximum of the nova appears to be well covered by the observations.

pg $m_{max} = 8.0$.

An objective-prism spectrogram of this nova obtained by the Harvard Observatory is the earliest photographic record of a supernova spectrum. Its original classification as an R-type spectrum, maintained until 1936, has been the cause of much confusion. For a description see *Harvard Ann.*, **76,** 37, 1916, and C. Payne Gaposchkin (*Ap. J.*, **83,** 173, 1936). An excellent description of the later visual spectrum of the nova was given by W. W. Campbell (*ibid.*, **5,** 233, 1897).

NGC 5457 Sc SS Ursae Majoris $m - M = 23.8$

The Heidelberg plates have been re-estimated by K. Reinmuth. A determination of the magnitudes of his comparison stars on the

international system resulted in the following magnitudes of the nova.

1909	Jan. 26	13.3	
	Feb. 21	13.3	
	Mar. 14	13.3	
	Apr. 8	13.2	
	May 9	14.1	
	Aug. 7	15.8	
1910	Mar. 8	16.9	
	Mar. 11	16.8	Hubble (*Mt. W. Contr.*, No. 376, p. 25; *Ap. J.*, **69**, 127 n., 1929)

The observed part of the light-curve is most unusual, with a constant brightness extending over seventy days. It would be of great importance if the history of this nova could be traced back on existing plates into the later months of 1908.

NGC 6946 Sc $m - M = 25.3$

The following magnitudes of the nova were determined by Shapley (*Pub. A.S.P.*, **29**, 210, 1917).

1917 July 19, 14.6; July 27, 15.6; Aug. 16, 15.9

A slitless spectrum was obtained August 16, 1917, by Pease and Ritchey, who described it as "a strong continuous spectrum, crossed by what appears to be a series of bright bands" (*Pub. A.S.P.*, **29**, 211, 1917). The plate, which is of rather poor quality, has recently been re-examined by Humason and the writer. There is no doubt that it shows the typical supernova spectrum with the wide λ 4600 band as the dominant feature.

THE ABSOLUTE MAGNITUDE OF SUPERNOVAE AT MAXIMUM

From the data of the preceding compilation we find that the magnitudes at maximum are well established for only 4 or 5 supernovae. In the majority of the cases the data are incomplete, usually consisting of a small number of observations belonging in general to the descending branch of the light-curve. Since the curves of the few well-observed supernovae indicate that the gradient during the first thirty days after maximum is of the order of 0.1 mag. per day, the brightest observed magnitudes may be easily 1–2 mag. fainter than the corresponding maximum magnitudes, and some procedure must

be adopted to derive from the observed parts of the light-curves approximate values of the maxima.

The obvious method in the present case is to fit the observed magnitudes to a typical supernova light-curve and to read off the extrapolated maxima. Since good photographic light-curves are available

TABLE 2

THE FINAL PHOTOMETRIC DATA

NGC	Type	$m-M$	Obs. m_{max}	Extrapol. m_{max}	M_{max}	Remarks
224	Sb	22.2	7.2	.....	−15.0	S Andr
1003	Sc	26.8	12.8	.....	14.0	
2535	SBc	.....	.....	14.7	.....	
2608	SBc	.....	.....	11.0:	.....	
2841	Sb	.....	.....	.....	.....	Only one observation
4157	Sc	.....	.....	14.4	.....	
IC 4182	Sc	24.8	8.2	.....	16.6	
4273	Sc	26.7	14.4	.....	12.3	Virgo cluster
4303	SBc	26.7	.....	12.8	13.9	Virgo cluster
4321, No. 1	Sc	26.7	.....	11.9	14.8	Virgo cluster
4321, No. 2	Sc	26.7	.....	.....	.....	Virgo cluster; only one observation
4424	SBb pec.	26.7	.....	11.1	15.6	Virgo cluster
4486	Eo	26.7	.....	12.0	14.7	Virgo cluster
4527	Sc	26.7	.....	13.0	13.7	Virgo cluster
5236	Sc	24.8	.....	.....	.....	Maximum doubtful
5253	Irr.	.....	8.0	.....	.....	Z Cent
5457	Sc	23.8	.....	.....	.....	SS U Ma; maximum doubtful
6946	Sc	25.3	.....	12.9	−12.4	
Mean	.....	.....	.....	.....	−14.3	

for the two recent supernovae in IC 4182 and NGC 1003, their combined curve has been used as the standard. Extrapolated maxima were thus obtained for 8 supernovae having two or more observations. The reduction showed that, in the mean for these 8 supernovae, the first observation was obtained seventeen days after the maximum, corresponding to a decrease in brightness of the supernova of 1.6 mag.

The final data are collected in Table 2. We consider first the 10 supernovae for which m_{max} and the distance modulus are known and

for which individual values of the maximum absolute magnitude have been obtained.[8] These values[9] are given in the sixth column of Table 2, ranging from $M = -12.3$ to $M = -16.6$, with the mean

$$\bar{M}_{\max} = -14.3 \pm 0.42 \text{ (m.e.)},$$

and the dispersion $\sigma = 1.27$ mag.

It is hardly necessary to point out that the intrinsic dispersion of $M_{\max}$ will be smaller than the computed value, since the latter includes the dispersions due to the errors in the adopted $m_{\max}$ and $m - M$. Although it is difficult at present to give accurate estimates of $\sigma_{m_{\max}}$ and σ_{m-M}, both are probably of the order of 0.3–0.4 mag., which would reduce the dispersion in the absolute magnitudes of supernovae to

$$\sigma_{M_{\max}} \approx 1.1 \text{ mag}.$$

Statistically we can derive $\bar{M}_{\max}$ in another way, for the relation $m_{\max} - m_{\text{neb}} = M_{\max} - M_{\text{neb}}$ permits us to compare directly the mean absolute magnitude of the supernovae at maximum with the mean absolute brightness of the extragalactic systems. Since the latter quantity is known, $M_{\max}$ is obtained at once.

The integrated photographic magnitudes of the nebulae in our list are collected in Table 3. The column headed (1) contains the magnitudes derived at the Harvard Observatory (Shapley-Ames, Seyfert, and others); (2), photoelectric magnitudes by Stebbins and Whitford, reduced to the international system; and (3), magnitudes derived by the writer, either from intrafocal exposures with a 5-inch Ross lens or from plates taken with a schraffierkassette on the 10-inch refractor of the Mount Wilson Observatory. The photographic magnitude of the very difficult system IC 4182, which has an ex-

[8] Since only magnitude differences are involved in this procedure, the latitude effect is unessential.

[9] The large percentage of cases in Table 2 of novae in members of the Virgo cluster needs perhaps a word of comment. It is a typical case of selection. (1) Among the nebulae of the northern hemisphere brighter than magnitude 13, the Virgo cluster and its extensions play numerically a large role. Many of its members were photographed with the Crossley reflector (Curtis' 3 novae). (2) The cluster is not far from the ecliptic and was covered in parts at least every year by the Heidelberg asteroid plates (2 novae by Wolf-Reinmuth). Finally the systematic search for supernovae in the cluster, carried out at Mount Wilson during recent years, added a sixth nova (Hubble-Moore).

tremely low surface brightness, has been derived from an exposure taken by F. Zwicky at the 18-inch Schmidt telescope of the Palomar Observatory. On his film the oscillating motion of a schraffierkassette has been imitated by a series of short overlapping extrafocal exposures which give for each star a square image of uniform den-

TABLE 3

INTEGRATED APPARENT MAGNITUDES OF NEBULAE

NGC	(1)	(2)	(3)	m_{neb} (Adopted)
224	.....	4.5	.....	4.5
1003	13.7	.....	13.1	13.1
2535	13.6	.....	13.8	13.7
2608	12.9	.....	13.6	13.6
2841	10.5	10.6	.....	10.6
4157	12.0	.....	.....	12.0
IC 4182	.....	.....	13.5*	13.5
4273	12.2	12.6	.....	12.4
4303	10.4	10.4	.....	10.4
4321	10.6	10.5	10.4	10.5
4424	12.6	12.5	.....	12.5
4486	10.7	10.2	10.1	10.1
4527	11.3	.....	.....	11.3
5236	8.0:	.....	8.8	8.8
5253	10.8	.....	11.0	11.0
5457	9.0	8.9	.....	8.9
6946	11.1:	11.1	.....	11.1

* From exposure by Zwicky with 18-inch Schmidt telescope on Palomar.

NOTES TO TABLE 3

(1) Harvard (Shapley-Ames, Seyfert, and others).
(2) Stebbins-Whitford.
(3) Baade (5-inch Ross intrafocal or schraffierkassette at 10-inch).

sity. The finally adopted magnitude for each nebula is given in the last column of Table 3.

The resulting values $m_{max} - m_{neb}$ for 14 supernovae are given in Table 4. Their mean value is

$$\overline{m}_{max} - \overline{m}_{neb} = + 0.34 \pm 0.66 \text{ (m.e.)} .$$

Hence, also,

$$\overline{M}_{max} = \overline{M}_{neb} + 0.34 .$$

To obtain $\overline{M}_{max}$ from this equation, we must establish the value of $\overline{M}_{neb}$. But it is not a priori clear which value for $\overline{M}_{neb}$ should be used: $M_{on} = -14.2$, which applies if we consider the nebulae contained in a given volume of space, or $\overline{M}_{n} = -15.2$, which is the proper value if the nebulae of our list present a selection made on the basis of apparent magnitude. In view of the small number of cases and the heterogeneous character of the discoveries, it seems hopeless to settle the question by general arguments. The data of Table 4,

TABLE 4

THE BRIGHTNESS OF SUPERNOVAE AT MAXIMUM COMPARED WITH THE INTEGRATED BRIGHTNESS OF THE SYSTEMS IN WHICH THEY APPEARED

NGC	m_{max}	m_{neb}	$m_{max}-m_{neb}$	M_{neb}
224	7.2	4.5	+2.7	−17.7
1003	12.8	13.1	−0.3	13.7
2535	14.7	13.7	+1.0	.....
2608	11.0:	13.6	−2.6	.....
4157	14.4	12.0	+2.4	.....
IC 4182	8.2	13.5	−5.3	11.3
4273	14.4	12.4	+2.0	14.3
4303	12.8	10.4	+2.4	16.3
4321, No. 1	11.9	10.5	+1.4	16.2
4424	11.1	12.5	−1.4	14.2
4486	12.0	10.1	+1.9	16.6
4527	13.0	11.3	+1.7	15.4
5253	8.0	11.0	−3.0	.....
6946	12.9	11.1	+1.8	−14.2
Mean	.....	.....	+0.34	−15.0

however, contain at least a partial answer to our question. Since for 10 of the 14 supernovae of our list the distance modulus is known, we can compute the absolute magnitudes of the nebulae in which they appeared. The data are given in the last column of Table 4. Their mean value, $\overline{M}_{neb} = -15.0$, indicates that for this group the nebulae of high luminosity do indeed predominate.[10] The inclusion of the

[10] Although a selection according to apparent magnitude is probably the main cause of the predominance of absolutely bright systems, another factor may play a role: that, other things being equal, the frequency of supernovae is a function of the stellar content (luminosity) of a system.

remaining four cases may reduce this value somewhat, since the considerable negative values of $m_{max} - m_{neb}$ for NGC 2608 and 5253 suggest systems for which the luminosity is below normal.[11] Probably we shall not be far from the truth if in the mean we count the four remaining nebulae as systems of normal brightness, $M_{on} = -14.2$. The mean absolute brightness for the whole group is then

$$\bar{M}_{neb} = -14.7 ,$$

and

$$\bar{M}_{max} = -14.4 \pm 0.47 \text{ (m.e.)} .$$

This value is in close agreement with that derived above from the systems of known distance modulus. Again the range in the observed values $m_{max} - m_{neb}$ indicates that the dispersions in the luminosities of the supernovae and in the extragalactic systems are nearly the same. The mean error, ± 0.47 of $\bar{M}_{max}$ has been computed on the assumption that such is the case from the mean error of the difference $\bar{m}_{max} - \bar{m}_{neb}$. As probably the best present value of the absolute photographic magnitude of supernovae at maximum, we adopt

$$\bar{M}_{max} = -14.3 \pm 0.4 ,$$

with the dispersion

$$\sigma_{M_{max}} \approx 1.1 \text{ mag} .$$

This value agrees so closely with the mean photographic luminosity, $\bar{M}_{on} = -14.2$, of the average extragalactic system that we can regard the two values as identical.

We have now the data required to place supernovae in their proper relation to common novae. Since for the latter

$$\bar{M}_{max} = -7.0,[12] \qquad \sigma_{M_{max}} = 0.53 ,$$

[11] For NGC 5253 the distance modulus derived from the distance-velocity relation points in the same direction.

[12] This value is the result of a new reduction of Hubble's observations by the writer. Hubble's original value is too small because he used an incorrect standard light-curve for the novae; cf. E. Hubble, *Pub. A.S.P.*, **50**, 107, 1938, and a forthcoming note by the writer.

their expected total range extends from $M = -5.4$ to $M = -8.6$, whereas the corresponding range for the supernovae extends from $\overline{M} = -11.0$ to $M = -17.6$.[13] In other words, the two groups are even statistically well separated from each other, with practically no chance for so-called "intermediate" cases. That the totally different spectra of common and supernovae leave no room for such hybrids has already been pointed out in the introductory paragraph. Finally, it is significant that no such case is indicated by the present material, although an incomplete light-curve might simulate that of a nova of intermediate luminosity. In this respect Lampland's nova in NGC 5236 and SS U Ma in Messier 101 are of special interest. They have been omitted from the foregoing discussions because their unusual light-curves made it difficult to decide whether the constant brightness at which they were discovered represents the maximum or an extended halt on the descending branch of the curve. In the former case both were supernovae of low luminosity, with $M = -10.8$ for Lampland's nova and $M = -10.5$ for SS U Ma, still close enough to the predicted lower limit for supernovae. So there is no doubt that both were supernovae. Moreover, it is also to be expected that supernovae of such low luminosity should have been found in two of the nearest extragalactic systems.

THE PREPONDERANCE OF SUPERNOVAE IN LATE-TYPE SPIRALS

In concluding the discussion of our present material, attention should be called to the curious fact that most of the known supernovae have appeared in late-type spirals (see Table 2, col. 2). In percentages we have for the types of system in which supernovae appeared:

Sc and SBc	72 per cent
Sb and SBb	16
Others	12

The unusual preponderance of late-type spirals in this list is quite in contrast with what we find for the sky at large. Since a complete classification is available now for the nebulae of the Shapley-Ames

[13] It would be difficult to represent graphically the two frequency distributions of common and supernovae on a proper scale because, if we take the Andromeda nebula as an example, the absolute frequencies of the two groups differ by a factor of 10,000.

list, brighter than magnitude 12.9 and north of declination $-30°$, they offer the most homogeneous material for a comparison. The corresponding figures, based on a total of 761 nebulae, are:[14]

Sc and SBc..... 32.1 per cent
Sbc and SBbc..... 10.9

Even if we include those cases which have been classified as intermediate between the Sb and Sc systems, the percentage of late-type spirals among the extragalactic systems brighter than 12.9 mag. is only 43 per cent, whereas 72 per cent of the nebulae in which supernovae have appeared belong to this type. In view of the scanty material it is difficult to decide at present whether this predominance of late-type spirals as source of supernovae has physical significance. Judgment therefore should be postponed until a sufficient number of cases is known.

THE NOVA B CASSIOPEIAE AND THE CRAB NEBULA

Nothing is known at present about the final state of supernovae. Indeed, it would require a supernova in our own galaxy to obtain this information. Fortunately, we know two objects in our galaxy which very probably have been supernovae and which may provide an answer to our question: B Cassiopeiae and the Crab nebula.

B Cassiopeiae, the bright nova of 1572, has often been cited as a possible supernova because of its unusual brightness at maximum. Since its position is sufficiently well known from Tycho Brahe's observations, repeated attempts have been made to identify it, but so far without success. Of these attempts, one should be mentioned here because the result obtained makes it improbable that B Cassiopeiae was a common nova. In 1922 Humason and Lundmark[15] investigated the spectra of all stars in the neighborhood of the computed position down to the fourteenth magnitude. Nothing unusual was found.[16] The importance of this negative result became clear

[14] These data were obtained by Dr. E. Hubble, who kindly permitted me to use them in this discussion.

[15] *Pub. A.S.P.*, **35**, 109, 1923.

[16] Lundmark was inclined to identify an M-type star close to Tycho's position with the nova because at least one other old nova, T Coronae Borealis, seemed to have an M-type spectrum. However, the M-type spectrum of T Coronae is most unusual, showing the high excitation line λ 4686 in emission (cf. Adams and Joy, *ibid.*, **33**, 263,

when recently Humason was able to show[17] that 16 former novae of our galaxy which he had investigated now have, without exception, very early B- or O-type spectra. Since no such spectrum has been found among the stars brighter than the fourteenth magnitude in the vicinity of B Cassiopeiae, we must conclude that, if this object was a common nova, its present brightness is fainter than $m = 14$. Since at maximum the nova reached the apparent brightness -4 to -5, the amplitude of its light-variation would then have been 18–19 mag. at least. This unusual amplitude is perhaps the strongest indication that B Cassiopeiae was not a common nova. For an investigation of such novae shows that their mean amplitude is of the order of 9 mag., with a maximum value of 14 for Nova Persei (1901), which also reached the upper limit of luminosity for common novae.

To extend the search to fainter magnitudes I have repeatedly photographed the field of B Cassiopeiae on blue- and red-sensitive plates. The only outstanding object of small color index which I have been able to locate is a star of photographic magnitude 17.0 and of a color index close to zero, in the position $\alpha = 0^h20^m18^s$, $\delta = +63°47'.1$ (1937). It is improbable, however, that this star is the lost B Cassiopeiae. Its distance of 5′ from the computed position is considerably larger than the error one would attribute to Tycho's position.[18] Moreover, the spectrum obtained by M. L. Humason showed that the star is a normal A0. Since this star of zero color index appeared as an outstanding object when the blue and the red plates were intercompared at the blink comparator, we must conclude that no other star of negative or zero color index exists in the close vicinity of B Cassiopeiae brighter than $m_{pg} = 18$, the limit of the search. This would increase the amplitude of B Cassiopeiae to more than 22 mag., an improbable figure for a common nova, as already pointed out. One might object that the nova has escaped

1921). The conclusion seems inevitable that T Coronae has a close faint companion of high temperature and that this companion is identical with the former nova. The unusual light-curve of the nova, which, after a steep descent, soon reached constant brightness, quite agrees with this assumption.

[17] *Mt. W. Contr.*, No. 596; *Ap. J.*, **88**, 228, 1938.

[18] The star cannot have a large proper motion since two pairs of plates taken with the 60-inch, separated by a time interval of fifteen years, did not reveal any motion when compared in the blink comparator.

detection because of a large color excess. But, even if we attribute to B Cassiopeiae the maximum luminosity of common novae, its distance would be of the order of only 100 parsecs, a value too small to be associated with a large amount of selective absorption. Altogether, the present evidence strongly suggests that Tycho's star was not a common nova. The most promising method for a further search seems to be a spectroscopic survey which would extend the investigation of Lundmark and Humason to very much fainter limits. It is intended to carry out this program as soon as an efficient slitless spectrograph for the Newtonian focus of our reflectors is available.

In many respects the Crab nebula seems to make a still better case of being a former supernova. The identification of this nebula with a nova observed by the Chinese in A.D. 1054 has been discussed so often that only the bare facts, leading to this conclusion, need to be repeated here.[19] The nebula expands, the transversal component of this motion amounting to $0''.18$ per year, the radial component to 1300 km per second. Since the present dimensions of the nebula place the beginning of the expansion back eight hundred years, to about A.D. 1100, the time of the outburst agrees with the appearance of the nova within the errors of the extrapolation. Moreover, the nova was observed "southeast of ζ Tauri, but near," i.e., in the field of the Crab nebula. Therefore, both position and time of the outburst strongly suggest that the Crab nebula is to be identified with the former nova. But in that case the nova very probably was a supernova. From the observed transversal and radial components of the expansion we obtain for the distance of the Crab nebula 1500 parsecs, corresponding to a modulus $m - M = 10.9$. Nothing is known about the observed brightness of the nova of A.D. 1054. The fact that it was visible for about six months is, however, sufficient to give at least a lower limit for the apparent brightness of the nova. For, since common and supernovae decrease in brightness during the first six months by 5–6 mag., and since the observations were made with the naked eye, the maximum apparent brightness of the nova must have been of magnitude 0 to -1 at least. This corresponds to

[19] The whole question has been thoroughly discussed by N. U. Mayall, *Pub. A.S.P.*, **49**, 101, 1937. See also O. Morgenroth, *Die Sterne*, **17**, 255, 1937.

$M = -11$ or $M = -12$ as the lower limit of the absolute brightness of the nova, a value high enough to place the star among the supernovae.[20]

The identification of the Crab nebula with a former supernova would be acceptable for another reason. It is well known that this nebula plays a rather unique role among the known emission nebulae of our galaxy. Because of its spectral characteristics it has sometimes been classified with the planetary nebulae. It cannot be a true member of this group, however, because its velocity of expansion is much too large for a planetary nebula. More recently it has been grouped with the nebulous envelopes surrounding Nova Persei (1901) and Nova Aquilae (1918), with which it shares the large velocity of expansion. But now we face another difficulty, that the Crab nebula stands apart because of the unusually high luminosity (presumably the mass) of the shell. Indeed, it is more than doubtful whether the expanding nebulosities around the two novae just mentioned will be visible at all eight hundred to nine hundred years after their outbursts.[21] The identification of the Crab nebula with a former supernova would avoid this difficulty.

It is needless to emphasize that more convincing data than those now available are required before either B Cassiopeiae or the Crab nebula can be considered with certainty as being former supernovae.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
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[20] *Note added to the proofs:* Dr. K. Lundmark kindly informs me that the nova of A.D. 1054 was also recorded by the Japanese, who called it "as bright as Jupiter." Since the apparent brightness of Jupiter at mean opposition is -2.2 mag., this would lead to the absolute magnitude $M_{max} \sim -13.1$ for the nova at maximum. The new evidence leaves hardly any doubt that the Crab nebula is the remnant of a former supernova. The whole matter will be fully discussed by Dr. Lundmark in a forthcoming paper.

[21] It is significant that no nebular envelopes suggesting old novae like Nova Persei (1901) and Nova Aquilae (1918) have ever been found in the extensive photographic surveys of the Milky Way. In view of the high frequency of common novae in our galaxy, the conclusion seems inevitable that the visibility of these envelopes is short, compared with the time necessary to produce an accumulation of such cases.

A PHOTOELECTRIC LIGHT-CURVE OF EROS

F. E. ROACH and LAURENCE G. STODDARD

ABSTRACT

Photoelectric observations of Eros made on February 4–5, 1938, when the earth was in or near the asteroid's equatorial plane, are presented. A new analysis of the form and size of Eros, following Krug and Schrutka-Rechtenstamm, leads to a three-axis ellipsoid with axes of 35.0, 15.6, and 7.2 km.

In a recent discussion of the photometric data of Eros, Watson[1] predicted that the earth would be in a plane 90° from Eros' pole of rotation on about February 5, 1938. Independent of any hypothesis of the geometrical figure of the asteroid and the law of reflection, the range of light variation should be a maximum at this time, the value predicted by Watson being 1.5 mag.

Eros' period of rotation is about 5^h16^m. In this interval there occur two maxima and two minima. Our photoelectric observations, starting 1938, Feb. $5^d4^h24^m$ G.C.T., cover just one-half of the rotation period, and include a light maximum at 5^h07^m and a minimum at 6^h21^m. The observed range is 1.38 mag. Since the color index is a constant equal to about +0.80 mag. throughout the rotation period[2], this range is the same as would have been obtained visually.

THE PHOTOMETER

The photoelectric cell of our photometer is a Kunz quartz cell having a sensitive surface of potassium hydride. The photoelectric current is amplified by a Pliotron FP-54 tube in a balanced circuit similar to that developed by DuBridge and Brown.[3] Characteristics of the photometer and amplifier are assembled in Table 1. The cell characteristics are for an anode potential of 300 volts, although it is possible to use somewhat higher voltages with correspondingly greater sensitivity.

The instrument was made, under the senior author's supervision, by two graduate students of the University of Arizona, Messrs.

[1] *Harvard Circ.*, No. 419, 1937.

[2] *Ibid.*, p. 12.

[3] *Rev. Scientific Instruments*, 4, 537, 1933.

Amos H. Hoff and Willard G. Schmidt. It may be used at any one of the three foci of the 36-inch reflector. The observations here reported were made at the Newtonian focus.

TABLE 1

PHOTOMETER AND AMPLIFIER CHARACTERISTICS

Galvanometer	Leeds and Northrup, Type H. S.
Current sensitivity	2.1×10^{-10} amps/mm at 1 meter
Amplifier:	
Grid leak resistance	5.65×10^{10} ohms
Current amplification	7.9×10^{5}
Resultant current sensitivity	2.66×10^{-16} amps/mm
Photoelectric cell:	
Anode potential	300 volts
Stellar magnitude corresponding to 1 mm deflection	14.4
Total light of 14.4 mag. as collected by 36-inch reflector	9.65×10^{-12} lumens
Light sensitivity of cell	27.5 microamperes/lumen

THE OBSERVATIONS

The observational data are recorded in Table 2. At the start there was in the western sky a five-day-old moon, which set at 5^h50^m G.C.T. Nine sky background readings were made, and the recorded readings in Table 2 have been corrected for this. The comparison star was HD 34020 (BD+21°800). Between the fifth and sixth observations the anode voltage was increased. The maximum difference in the secants of the zenith distances of the asteroid and the comparison star was 0.01. This would lead to maximum differential extinction of about 0.002 mag. In the reductions differential extinction has not been applied. Alternate settings were made on the comparison star and on the asteroid, with four galvanometer readings per setting. The times in Table 2 refer to the observations of Eros. The comparison star readings of the third column have been taken from settings both before and after the Eros readings, each corrected for the rapidly changing sky background. The results are plotted in Figure 1.

TABLE 2

THE OBSERVATIONS

TIME (G.C.T.) FEB. 5, 1938	GALVANOMETER DEFLECTION (MM.)		Δ mag	m_{pg} OF EROS (m_{pg} HD 34020 = 9.70)
	Eros	HD 34020		
4^h24^m	18.6	50.1	1.08	10.78
4 39	23.5	49.2	0.80	10.50
4 59	30.3	49.05	0.52	10.22
5 19	30.6	48.7	0.50	10.20
5 31	25.3	47.8	0.69	10.39
5 51	16.0	49.65	1.23	10.93
6 07	10.4	44.55	1.58	11.28
6 20	8.4	45.75	1.84	11.54
6 31	9.2	45.2	1.73	11.43
6 41	12.9	43.2	1.31	11.01
6 50	13.1	39.15	1.19	10.89

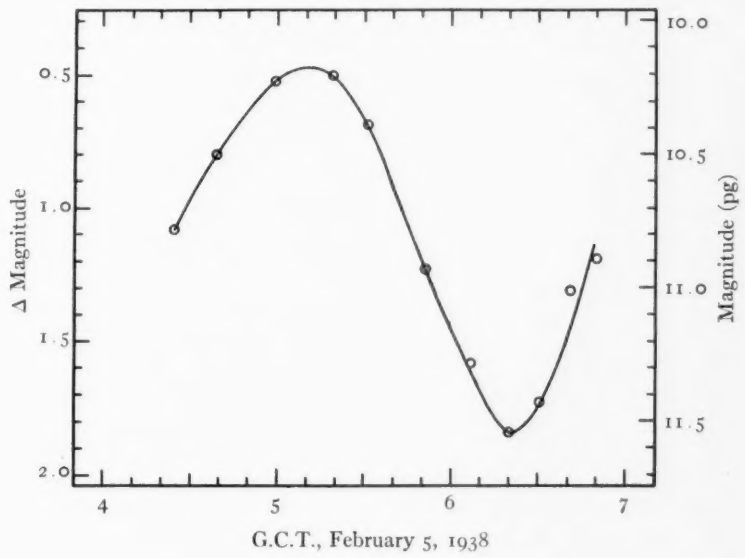


FIG. 1

ANALYSIS OF OBSERVATIONS

In a recent paper[4] Krug and Schrutka-Rechtenstamm have given an analysis of the form and size of Eros based upon the following assumptions: (*a*) the object is a three-axis ellipsoid at full phase rotating about its shortest axis; (*b*) it reflects according to Lambert's law. They obtained equations giving the brightness in terms of the ellipsoid's semiaxes, a, b, and c, where $a > b > c$. We reproduce here their equation (1):

$$\left.\begin{aligned} Q &= Q_A \cos^2 A + Q_B \cos^2 B + Q_C \cos^2 C \,, \\ &= Q_C \cos^2 C + \sin^2 C[Q_A \cos^2 (\omega t + \phi) + Q_B \sin^2 (\omega t + \phi)] \,, \end{aligned}\right\} \tag{1}$$

where Q is the observed brightness; Q_A, Q_B, and Q_C are the brightnesses as seen along the a, b, and c axes, respectively; A, B, and C are the angles between the line of sight and the a, b, and c axes, respectively; ω is the angular velocity of rotation; t is the time; and ϕ is a phase angle. At maximum and minimum light this becomes

$$\left.\begin{aligned} Q_{\max} &= Q_C \cos^2 C + Q_B \sin^2 C \,, \\ Q_{\min} &= Q_C \cos^2 C + Q_A \sin^2 C \,. \end{aligned}\right\} \tag{2}$$

If $C = 90°$, i.e., if the observer is in the equatorial plane of the ellipsoid,

$$\left.\begin{aligned} Q &= Q_A \cos^2 (\omega t + \phi) + Q_B \sin^2 (\omega t + \phi) \,, \\ Q_{\max} &= Q_B \,, \\ Q_{\min} &= Q_A \,. \end{aligned}\right\} \tag{3}$$

Equations (1), (2), and (3) apply to any geometrical figure at full phase reflecting according to Lambert's law. A least-squares solution of equation (3) for our data gives $Q_A = 1$, $Q_B = 3.60$, $\omega = 0.0213$ radians per minute, and $\phi = 0$ if $t = 0$ at 3^h52^m G.C.T. This solution is plotted in Figure 2, together with the observations.

Equations (2) in the paper by Krug and Schrutka-Rechtenstamm give the dependence of Q_A, Q_B, and Q_C on the semiaxes of a three-

[4] *Zs. f. Ap.*, **13**, Part I, 1, 1936.

axis ellipsoid. We wish to give an analysis based upon the ratios Q_B/Q_A and Q_C/Q_B, which follow at once from these equations, viz.,

$$\frac{Q_B}{Q_A} = \frac{a^2}{b(b^2 - c^2)}\left[\frac{\sqrt{a^2 - c^2}(b^2 - c^2) - (a^2 - c^2)bE + b(a^2 - b^2)F}{-b\sqrt{a^2 - c^2} + b^2E + (a^2 - b^2)F}\right], \quad (4)$$

$$\frac{Q_C}{Q_B} = \frac{b(a^2 - b^2)}{c^2}\left[\frac{b^2E - c^2F}{(b^2 - c^2)\sqrt{a^2 - c^2} - b(a^2 - c^2)E + b(a^2 - b^2)F}\right], \quad (5)$$

where

$$E = \int_0^{\phi} \sqrt{1 - k^2 \sin^2 z}\, dz\,,$$

$$F = \int_0^{\phi} \frac{dz}{\sqrt{1 - k^2 \sin^2 z}}\,,$$

$$k = \frac{a\sqrt{b^2 - c^2}}{b\sqrt{a^2 - c^2}}\,,$$

$$\phi = \text{arcosine}\,\frac{c}{a}\,.$$

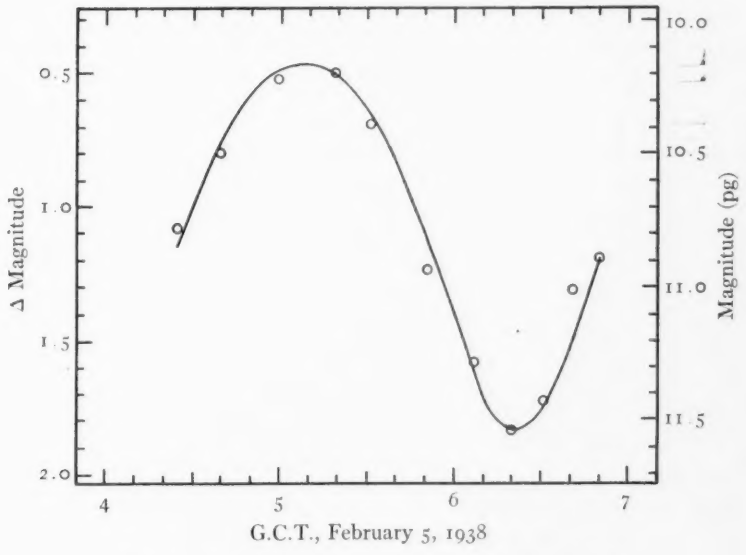


FIG. 2.—Plot of least-squares solution of equation (3)

The functions E and F with k and ϕ as arguments may be found in *Formules et Tables Numériques* by L. Potin.

Krug and Schrutka-Rechtenstamm have utilized forty-four observations of the magnitudes and light variations of Eros and have made a least-squares solution for (1) the position of the north pole of rotation; (2) the values of Q_A, Q_B, and Q_C; and (3) the corresponding values of a, b, and c. A summary of their results follows: Position of the north pole of rotation: $\alpha = 22^h11^m6$, $\delta = +47^\circ.7$; $Q_A = 0.827$; $Q_B = 2.168$; $Q_C = 2.830$; $2a = 34.6$ km; $2b = 19.3$ km; $2c = 16.3$ km. From these results we find $Q_B/Q_A = 2.622$, corresponding to a maximum light range of 1.046 mag.; $Q_C/Q_B = 1.305$; if $a = 1$, $b = 0.558$ and $c = 0.471$.

A criticism of these results which was pointed out by Krug and Schrutka-Rechtenstamm is that the maximum amplitude of light variation permitted by the model is definitely lower than is observed as the earth approaches the equatorial plane, i.e., as C approaches 90°. For example, at the time of the observations reported in this paper the value of C was 84°. With this value of C their solution gives for the range of light variation 1.025 mag., as compared with the observed 1.38 mag. of this paper.

We have made a new analysis of the data of Krug and Schrutka-Rechtenstamm on the following basis: (1) In their table of observations we have omitted all observations in which no variation was observed; (2) we assume $Q_B/Q_A = 4.00$, corresponding to $\Delta m = 1.50$ mag., and solve for Q_C/Q_B; (3) we assume their pole of rotation; (4) this analysis gives no weight to the *actual* magnitude of any observation but only to the range of light variation. The latter is doubtless known with greater accuracy than the former. The analysis of Krug and Schrutka-Rechtenstamm gave weight to both the actual magnitude and the range. A least-squares solution gives $Q_C/Q_B = 3.25$.

A simultaneous solution of equations (4) and (5), assuming $a = 1.00$, yields $b = 0.447$ and $c = 0.207$. If $2a$ is taken as 35 km[5] this gives $2b = 15.6$ km and $2c = 7.2$ km. Our solution gives for $C = 84^\circ$ a range of 1.40 mag., as compared with the 1.38 mag. observed.

[5] Cf. Watson, *op. cit.*, p. 9.

Two difficulties have always appeared in previous efforts to analyze the observations of Eros. First, computations of the phase coefficient have shown a wide variation from one opposition to another. Second, the absolute magnitudes derived at various oppositions have shown a decided lack of agreement. These two constants are not independent, as the value of the absolute magnitude in any given case depends on the assumed phase coefficient. Our model

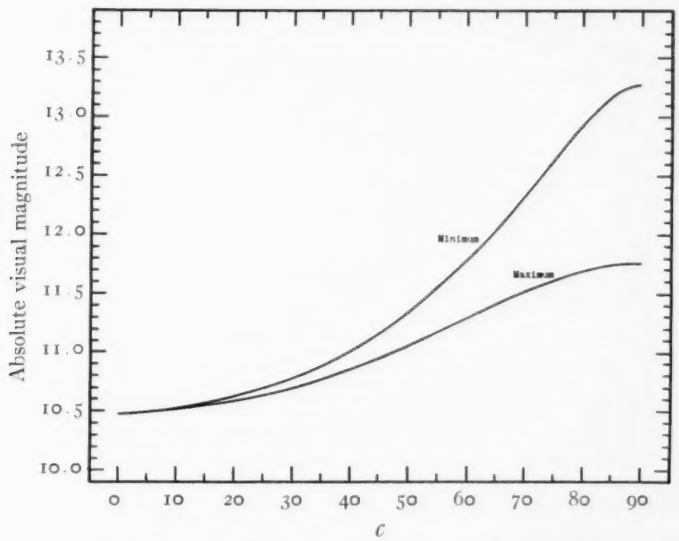


FIG. 3.—Predicted absolute visual magnitude of Eros. The abscissa is the angle between the line of sight and the axis of rotation.

gives a mechanism for the possible clearing-up of these enigmas, as does the model of Krug and Schrutka-Rechtenstamm in a lesser degree. In the first place, there will be no single value of the absolute magnitude, but rather a variation from maximum brightness, if the observer is in the direction of the pole of rotation, to a minimum value when $C = 90°$. Similarly, the phase coefficient will depend upon the rate at which C changes during a series of observations.

In Figure 3 we show a plot of the absolute visual magnitudes at maximum and minimum, on the basis of our solution. This curve depends upon the assumptions that the photographic magnitude

of HD 34020 is 9.70 mag., that the color index of Eros is + 0.80 mag., and that the phase coefficient is 0.022 mag. per degree of phase angle. The variation of the predicted range with the angle C is shown in Figure 4.

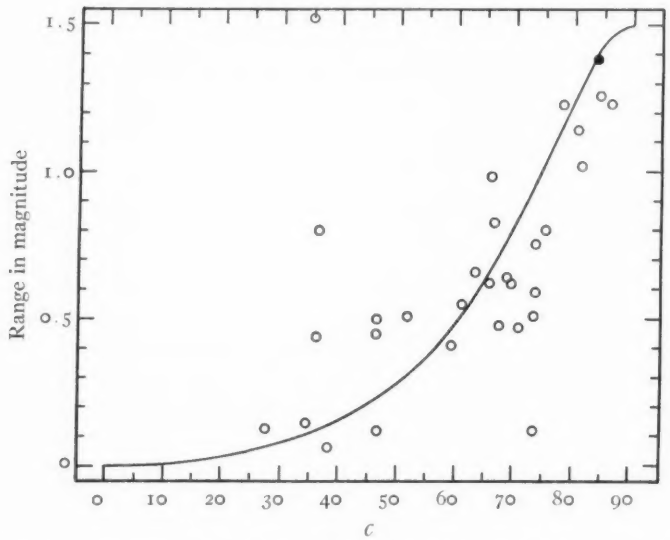


FIG. 4.—Predicted variation of light range with the angle C. The open circles represent the data of Krug and Schrutka-Rechtenstamm. The solid circle represents the data of this paper.

CONCLUSION

It should be remarked that this analysis assumes Lambert's law of reflection, and that Eros is at full phase. The second of these assumptions is definitely not true for our observations, since the phase angle was 46°44′. The fair agreement of our observations and equation (3) may, therefore, be fortuitous.

STEWARD OBSERVATORY
UNIVERSITY OF ARIZONA
TUCSON, ARIZONA
May 23, 1938

PHYSICAL PROCESSES IN GASEOUS NEBULAE

IV. THE MECHANISTIC AND EQUILIBRIUM TREATMENT OF NEBULAR STATISTICS

DONALD H. MENZEL, LAWRENCE H. ALLER,
AND JAMES G. BAKER[1]

ABSTRACT

The statistical equilibrium of nebulae has been discussed in the past from two points of view. H. H. Plaskett employed a *mechanistic* argument, where the captures in a given level are subdivided according to the probability of transitions to lower levels and are followed through successive transitions down to the ground level. Carroll criticized Plaskett's treatment and developed equations on an *equilibrium* argument. We have shown that the two points of view, when properly applied, lead to the same result; and we have derived the necessary equations, which will lead to a determination of electron temperature.

There are essentially two methods for approaching the problem of the distribution of atoms among the various atomic states of a hydrogen nebula. The first one, historically, used by H. H. Plaskett,[2] involves counting the number of captures in a given atomic level and then tracing the subsequent history of these captured electrons as they cascade to lower atomic levels, some going to one and some to other levels. This point of view, which may be termed the "mechanistic," was criticized by Carroll,[3] who argued that it did not insure statistical equilibrium. In our opinion the criticism is unjustified.

Carroll took what might be called the "equilibrium" point of view, equating the number of atoms arriving in a given level to the number leaving it, without inquiring into the past or future history of a given transition. We shall show later in this paper that the two points of view are essentially equivalent. Carroll gave a rigorous solution for an atom possessing seven discrete levels and a continuum. Plaskett, using the mechanistic argument in calculating for a seven-level atom, was able to allow in part for the equations neglected by Carroll. His results should be, therefore, in closer agreement with

[1] Society of Fellows, Harvard University.

[2] *Harvard Circ.*, No. 335, No. 1928.

[3] *M.N.*, **90**, 588, 1930.

the exact theory than the later ones of Cillié,[4] derived by the method of equilibria.

In calculating the partition of atoms in higher levels, we have found it more convenient to apply the equilibrium approach. But for certain problems the mechanistic point of view is simpler. In evaluating the temperature T_e of the electron gas, for example, in terms of the temperature T_1 of the exciting radiation, one may equate mechanistically the number of ionizations from the ground level to the number of captures on all levels. In thermodynamic equilibrium, detailed balancing would ensue and T_e must equal T_1. One of us has shown[5] that the same equality of electron and radiation temperatures would have to exist in an atom with one discrete state, whatever the dilution of the radiation. But when many or an infinite number of levels are considered, the temperatures will not be equal, and $T_e < T_1$. In the present paper we shall neglect collisional excitation of all kinds, although such an effect could be easily included. Our aim is to prove that the mechanistic and equilibrium equations are consistent with one another, and that they may be used interchangeably in various types of calculations.

Four fundamental equations of equilibrium may be written down at once in the functional notation of I,[6] and in the form presented below they are symbolically exact. The first equation is that of statistical equilibrium for the ground level, that is to say, it describes the physical condition that the number of atoms entering the ground level per unit time and volume, by all possible routes, must be precisely equal to the number leaving the ground level:

$$\sum_2^\infty F_{n1} + \int_{\nu_1}^\infty F_{\kappa 1} d\nu = \sum_2^\infty F_{1n} + \int_{\nu_1}^\infty F_{1\kappa} d\nu \,. \qquad (1)$$

Next in order is the equation of statistical equilibrium for the excited level, n. This equation appeared as I (31), and was em-

[4] *Ibid.*, **92**, 820, 1932; **96**, 771, 1936.

[5] D. H. Menzel, *Pub. A.S.P.*, **43**, 334, 1931.

[6] D. H. Menzel, *Ap. J.*, **85**, 330, 1937; D. H. Menzel and J. G. Baker, *ibid.*, **86**, 70, 1937; J. G. Baker and D. H. Menzel, *ibid.*, **88**, 52, 1938.

ployed in a simplified form in II[6] and III[6] for determining the Balmer decrement. Physically, this equation describes the condition that the number of atoms entering an excited level, n, by all possible routes, must equal the number of atoms leaving that level per unit time and volume:

$$\left.\begin{aligned}\sum_{n+1}^{\infty} F_{n''n} + \int_{\nu_n}^{\infty} F_{\kappa n}\,d\nu + \sum_{1}^{n-1} F_{n'n} &= \sum_{n+1}^{\infty} F_{nn''} \\ &\quad + \int_{\nu_n}^{\infty} F_{n\kappa}\,d\nu + \sum_{1}^{n-1} F_{nn'}\,.\end{aligned}\right\} \tag{2}$$

The third equation of statistical equilibrium describes the physical condition that the number of atoms entering the continuum must be equal to the number of atoms leaving, per unit time and volume:

$$\sum_{1}^{\infty} \int_{\nu_{n'}}^{\infty} F_{n'\kappa}\,d\nu = \sum_{1}^{\infty} \int_{\nu_{n'}}^{\infty} F_{\kappa n'}\,d\nu\,. \tag{3}$$

The last fundamental equation first appeared as I (33) and is rewritten below in a form in which E is replaced by $Fh\nu$. This equation is a description of the physical condition for radiative equilibrium, namely, that the amount of energy absorbed must equal the amount of energy lost per unit time and volume:

$$\left.\begin{aligned}&\sum_{1}^{\infty} \int_{\nu_n}^{\infty} F_{\kappa n} h\nu_{\kappa n}\,d\nu + \sum_{2}^{\infty} \sum_{1}^{n-1} F_{nn'} h\nu_{nn'} + \int_{0}^{\infty} \int_{0}^{\infty} F_{\kappa''\kappa} h\nu_{\kappa''\kappa}\,d\nu_{\kappa''}\,d\nu_{\kappa} \\ &= \sum_{1}^{\infty} \int_{\nu_n}^{\infty} F_{n\kappa} h\nu_{n\kappa}\,d\nu + \sum_{2}^{\infty} \sum_{1}^{n-1} F_{n'n} h\nu_{n'n} + \int_{0}^{\infty} \int_{0}^{\infty} F_{\kappa'\kappa} h\nu_{\kappa'\kappa}\,d\nu_{\kappa'}\,d\nu_{\kappa}\,.\end{aligned}\right\} \tag{4}$$

For the gaseous nebulae exposed to dilute temperature radiation with a dilution factor comparable to that found in planetaries, certain of the terms in the foregoing general equation may be neglected. By far the greater proportion of neutral atoms are in the ground level. The problem of radiative transfer becomes important

only for the Lyman lines and Lyman continuum. Even with the second level there are too few atoms for a Balmer transfer problem to exist. In other words, the nebula is relatively transparent to the Balmer radiation. The number of atoms entering level n by absorption of radiation in a subordinate line will be negligibly small in comparison with the number cascading down from higher levels or being excited from the ground level. In view of the small population in the upper levels and of the dilution of the radiation, photoelectric ionization from all levels other than the first can also be safely neglected. Omitting all terms involving upward transitions from $n \geqslant 2$, and in the continuum, we can re-write the four fundamental equations in a simplified form:

$$\sum_{2}^{\infty} F_{n1} + \int_{\nu_1}^{\infty} F_{\kappa 1} d\nu = \sum_{2}^{\infty} F_{1n} + \int_{\nu_1}^{\infty} F_{1\kappa} d\nu \,, \tag{5}$$

$$\sum_{n+1}^{\infty} F_{n''n} + \int_{\nu_n}^{\infty} F_{\kappa n} d\nu + F_{1n} = \sum_{1}^{n-1} F_{nn'} \,, \tag{6}$$

$$\int_{\nu_1}^{\infty} F_{1\kappa}\, d\nu = \sum_{1}^{\infty} \int_{\nu_n}^{\infty} F_{\kappa n} d\nu = \sum_{2}^{\infty} \int_{\nu_n}^{\infty} F_{\kappa n} d\nu + \int_{\nu_1}^{\infty} F_{\kappa 1} d\nu \,, \tag{7}$$

$$\left.\begin{aligned} \sum_{1}^{\infty} \int_{\nu_n}^{\infty} F_{\kappa n} h\nu_{\kappa n} d\nu + \sum_{2}^{\infty} \sum_{1}^{n-1} F_{nn'} h\nu_{nn'} + \int_{0}^{\infty} \int_{0}^{\infty} F_{\kappa\kappa'} h\nu_{\kappa\kappa'} d\nu_{\kappa} d\nu_{\kappa'} \\ = \int_{\nu_1}^{\infty} F_{1\kappa} h\nu_{1\kappa} d\nu + \sum_{2}^{\infty} F_{1n} h\nu_{1n} \,. \end{aligned}\right\} \tag{8}$$

To this set of equations we now add a fifth equation, which may be regarded as mechanistic in nature. The amount of energy gained by the continuum by ionization from the ground level reappears in emission by captures on all the levels of the atom and subsequent cascading to the ground level, and also in free-free emissions. The number of atoms captured in level n per unit time and volume must be equal to the number contributed by level n to the ground level per

unit time and volume by downward transitions with a net energy loss of amount $h\nu_{n1}$. Consequently, we have

$$\left.\begin{aligned}\int_{\nu_1}^{\infty} F_{1\kappa} h\nu_{1\kappa} d\nu &= \sum_{1}^{\infty} \int_{\nu_n}^{\infty} F_{\kappa n} h(\nu_{\kappa n} + \nu_{n1}) d\nu + \int_0^{\infty}\int_0^{\infty} F_{\kappa\kappa'} h\nu_{\kappa\kappa'} d\nu_\kappa d\nu_{\kappa'} \\ &= \sum_{1}^{\infty} \int_{\nu_n}^{\infty} F_{\kappa n} h\nu_{\kappa n} d\nu + \sum_{2}^{\infty} \int_{\nu_n}^{\infty} F_{\kappa n} h\nu_{n1} d\nu + \int_0^{\infty}\int_0^{\infty} F_{\kappa\kappa'} h\nu_{\kappa\kappa'} d\nu_\kappa d\nu_{\kappa'} .\end{aligned}\right\} \quad (9)$$

We may establish the aforementioned consistency of the mechanistic and statistical points of view very simply by manipulation of the preceding five equations. First, we sum (6) from 2 to infinity in order to render it independent of n, and then we obtain

$$\sum_{2}^{\infty}\sum_{i}^{\infty} F_{n''n} + \sum_{2}^{\infty} \int_{\nu_n}^{\infty} F_{\kappa n} d\nu + \sum_{2}^{\infty} F_{1n} = \sum_{2}^{\infty}\sum_{1}^{n-1} F_{nn'} . \quad (10)$$

The relationship

$$\sum_{2}^{\infty} \left\{ \sum_{n+1}^{\infty} F_{n''n} - \sum_{2}^{n-1} F_{nn'} \right\} = 0 \quad (11)$$

is easily proved by expanding the symbolic summations, which will be found to be identical. Each summation reduces to a series of the form

$$F_{32} + F_{42} + F_{52} + \ldots + F_{43} + F_{53} + F_{63} + \ldots + F_{54} + \ldots + \ldots$$

Hence, (10) becomes

$$\sum_{2}^{\infty} \int_{\nu_n}^{\infty} F_{\kappa n} d\nu + \sum_{2}^{\infty} F_{1n} = \sum_{2}^{\infty} F_{n1} . \quad (12)$$

Using (7), we obtain

$$\int_{\nu_1}^{\infty} F_{1\kappa} d\nu + \sum_{2}^{\infty} F_{1n} = \sum_{2}^{\infty} F_{n1} + \int_{\nu_1}^{\infty} F_{\kappa 1} d\nu , \quad (13)$$

which is (1). Therefore, equations (5), (6), and (7) are consistent with one another. We now must show that (8) and (9) are consistent with one another and with (5), (6), and (7). Upon substituting (9) into (8), we obtain

$$\left.\begin{aligned}\sum_{2}^{\infty}\sum_{1}^{n-1}F_{nn'}h\nu_{nn'} &= \sum_{2}^{\infty}\int_{\nu_n}^{\infty}F_{\kappa n}h\nu_{n1}d\nu + \sum_{2}^{\infty}F_{1n}h\nu_{n1}\\ &= \sum_{2}^{\infty}h\nu_{n1}\left[\int_{\nu_n}^{\infty}F_{\kappa n}d\nu + F_{1n}\right]\\ &= \sum_{2}^{\infty}h\nu_{n1}\left[\sum_{1}^{n-1}F_{nn'} - \sum_{n+1}^{\infty}F_{n''n}\right]\\ &= \sum_{2}^{\infty}\sum_{1}^{n-1}F_{nn'}h\nu_{n1} - \sum_{2}^{\infty}\sum_{n+1}^{\infty}F_{n''n}h\nu_{n1}\,,\end{aligned}\right\} \quad (14)$$

where (6) has been used.

We next replace $h\nu_{nn'}$ in (14) by $(h\nu_{n1} - h\nu_{n'1})$, and find that

$$\sum_{2}^{\infty}\sum_{2}^{n-1}F_{nn'}h\nu_{n'1} = \sum_{2}^{\infty}\sum_{n+1}^{\infty}F_{n''n}h\nu_{n1}\,. \quad (15)$$

A consideration of the uniform convergence of this expression will readily show that it is really an identity. As was indicated for (11), the double summations on both sides of (15) consist of the same elements. The five equations, (5), (6), (7), (8), and (9), are thus shown to be consistent, and the mechanistic and equilibrium points of view are reconciled. The next paper of this series will employ these equations to find the relationship between the excitation and electron temperatures of a nebula.

HARVARD OBSERVATORY

A RELATIONSHIP BETWEEN COLOR, SPECTRUM AND ABSOLUTE MAGNITUDE FOR G-TYPE STARS

JOHN S. HALL

ABSTRACT

The colors and magnitudes of 170 bright HD G-type stars have been determined with a photoelectric photometer, sensitive to the infrared, at the Sproul Observatory. A null method involving the use of two photocells was used to make 36 per cent of the observations. It is briefly described.

Accurate color excesses were formed from a combination of these data, Becker's colors, and Hertzsprung's photographic magnitudes with accurate spectral types. These color excesses are strongly related to the Victoria spectroscopic absolute magnitudes for stars of spectral class G2 to K0.

The presence of red secondary components to several close visual binaries is strongly suggested by (1) a comparison of the blue-violet with the red-infrared colors and (2) the unusually large differences (of the proper sign) between the Victoria and the colorimetric absolute magnitudes. The data suggest that two near-by stars not known to be binaries may have red companions.

The colors of stars of spectral types F8 to G1 of the main sequence are found to show very little cosmic dispersion. This conclusion confirms that very recently formulated by W. W. Morgan.

INTRODUCTION

This paper discusses colors and magnitudes of G-type stars observed with a photoelectric photometer attached to the 24-inch telescope of the Sproul Observatory. The stars included in this program are those classified as G0 or G5 in the *Henry Draper Catalogue*, brighter than 6.0 visual magnitude and included in the Mount Wilson list of spectroscopic absolute magnitudes[1] of 1935. With a few exceptions, stars known to be spectroscopic binaries and stars of southern declination have been omitted.

METHOD

The same filters were used for the execution of this program as were used for a similar program[2] involving HD K0 stars. These filters are Jena colored optical glasses BG17 and RG9, of 3- and 2-mm thicknesses, respectively. When a stop watch was used to time the rate of drift of the electrometer needle, six readings were made

[1] *Ap. J.*, **81**, 187, 1935. [2] *Ibid.*, **84**, 369, 1936.

with each filter in position. In order to check the sensitivity of the photometer with tilt and time, five readings were made with a standard lamp in position for each setting of the telescope.

Thirty-six per cent of the observations included in this paper were made by a null method employing two photocells. The light from the star struck the cathode of the caesium-oxide-silver cell in the usual way. The anode of this cell was connected to the cathode of a potassium vacuum cell as well as to the electrometer needle. The photo-currents from the two cells were therefore opposed to each other. Light from a 3.8-volt incandescent lamp (in series with the standard lamp) passed through an Eastman photographic wedge to the potassium cathode. The wedge was mounted on a carriage in such a way that the distance it had to be moved in order to balance these photo-currents could be accurately determined. When this point of balance was reached, the electrometer needle would remain stationary. The distance between wedge settings for each filter, multiplied by a constant, gives the observed color in magnitudes. The distance of the setting for RG9 from an arbitrary zero point is proportional to the magnitude of the star. Actually, the balance point with the standard lamp in position was usually also observed as a check on this arbitrary zero.

This null method has several advantages over the method by which the observer times the motion of the electrometer needle with a stop watch. It was found that two readings for each filter were sufficient to determine the color of each star with an accuracy equal to that resulting from six readings with the stop watch. The null method is somewhat faster and is much easier for the observer. The reductions are very simple. The balance photocell must have a cathode of very small dimensions in order that the quick response of the needle will be maintained. The addition of the particular cell I used reduced the response of the needle by 30 per cent. Also, background photo-currents must be kept negligibly small in comparison with photo-currents due to the stellar radiation.

The scale of the wedge as used with the photometer was determined by a laboratory method similar to that previously described.[2] A further check on the scale of the photometer when the null method was used is afforded by a comparison of the magnitudes of the

same stars observed in common by the two methods. The agreement is excellent. In order to be sure that the observations of the entire program were homogeneous, every star was observed at least once by the stop-watch method.

EFFECTIVE WAVE LENGTHS

In May, 1936, a few months after this program was started, the photocell which had been in use since 1933 was rendered useless as a result of an accident. A new one ordered soon after the mishap from the Bell Laboratories in New York, is still in active service. Spectrophotometric measures of the same bright stars with both cells indicate that their color-curves are very similar.

Figure 1 shows the response of the new cell as a function of the wave length for three stars of different color. The spectrum was formed by an objective grating, and the observations were made in the manner already described.[2] Although the observed response has been corrected to the zenith by means of observed extinction coefficients, it has not been corrected for small differential effects which may result from the fact that the starlight is reflected from two aluminized surfaces before striking the photocell. The response with each filter in position is shown for η Bootis. The effective wave length for BG17 and RG9 are at 6550 and 8300 A, respectively.

The corresponding wave lengths found for Capella with the former cell are 6710 A and 8230 A. Since the colors in the present paper are reduced to the same system as that for the colors of the Ko stars, these values may be regarded as the effective wave lengths of the colors of the G stars.

In the former paper the effective wave length of a Ko star of average color is given as 8300 A for the magnitudes determined with filter RG9. For the present program, since η Bootis is not so red as the star of average color, the corresponding figure for an average G star is near 8340 A.

When the relatively slow rate at which the energy of G and K stars changes with wave length in the infrared region is taken into consideration, these differences in the effective wave lengths for RG9 are very small. Recent observations of an Mb star, Beta Pegasi, yielded a value of 8510 A for RG9 on a night when the water-vapor

bands near 9400 A were weak. On another night, when the same bands were pronounced, this effective wave length was 8480 A. For

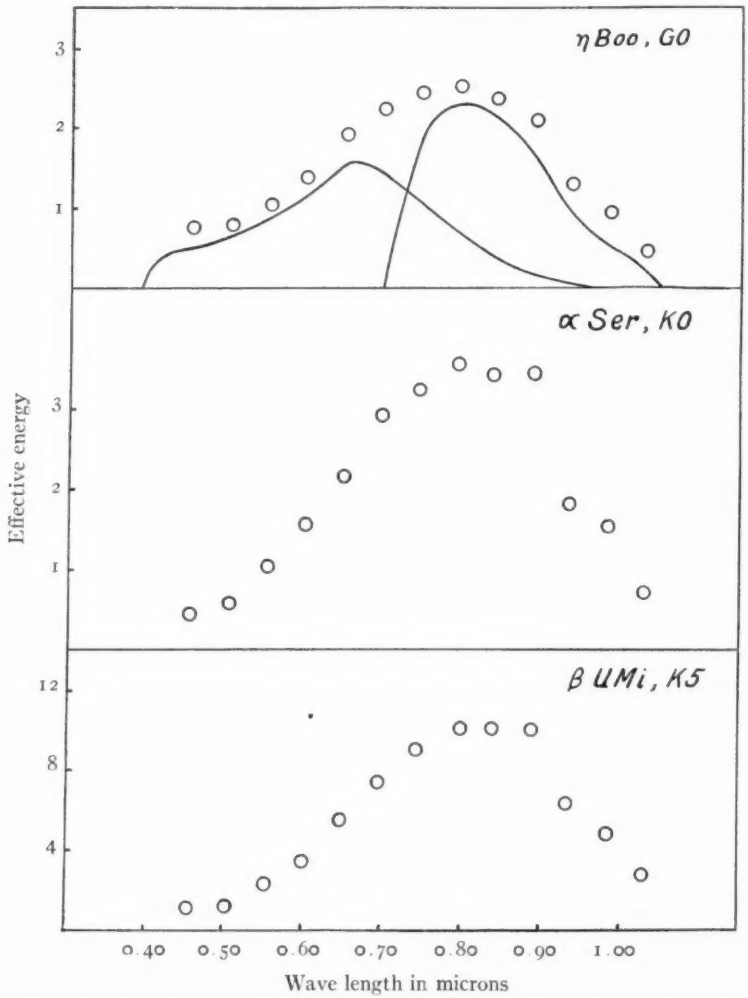


FIG. 1.—The effective energy or the rate of drift of the electrometer needle is given in units of tenths of an eyepiece division per second. Each of these stars has been measured on three different nights as part of a general spectro-photoelectric program. The mean rate of deflection is plotted for each wave-length band. The slit widths were such as to admit 483 A of the spectra at a time. For η Boo the transmissions of the filters have been multiplied by the ordinates of a smoothed curve in deriving the response for each filter as a function of the wave length.

many purposes, the magnitudes derived from observations with RG9 in position may be assumed to have the same effective wave length as that found by Hetzler[3] in his infrared surveys. Using Eastman 1-P plates with Wratten 87 filter, the value he finds for stars of similar color is near 8450 A. No error larger than 0.07 mag. should ever occur for G, K, or M stars when this assumption is made.

OBSERVATIONS AND REDUCTIONS

Color observations numbering 594 were made of 170 stars on thirty-five nights. In order to combine these results with those of the program involving the Ko stars, 97 observations were made on 28 of these stars. Infrared magnitudes of all these stars were determined from 674 comparisons of the effective stellar energy, through filter RG9, with that from an incandescent lamp. The current through this lamp was carefully controlled. Seventeen magnitude observations were discarded for various reasons while the reductions were in progress.

The program which involved HD Ko stars and the present program will be designated as the *K* and the *G* programs, respectively.

The same extinction coefficients were used to reduce the colors and magnitudes of the G stars to the zenith as were previously determined for the K stars. The extinction coefficient used for the colors was, therefore, −0.05 mag.; and for the magnitudes, −0.13 mag. Since almost all the stars in the present program are north of the equator and since they were observed near the meridian, these corrections were usually very small.

REDUCTION OF COLORS TO SYSTEM OF K PROGRAM

The reduction of the colors of the G stars was made in a manner similar to and independent of that of the K stars Fifty-eight constants were used to reduce the observations of thirty-five nights to a homogeneous system.

Two relationships were used to reduce the *G* system of colors to that of the *K* program. These are: from PGC 41 to 2626 inclusive, $K'=0.824\ G-0.185$ mag.; and for the other group, $K'=0.824\ G-0.163$ mag. The constant 0.824 was determined from a

[3] *Ibid.*, **86**, 510, 1937.

least-squares solution involving all 28 stars observed in common. It will be noted that in making this reduction I have assumed that a systematic difference in the zero points of the two systems of color for different halves of the sky is due entirely to a systematic error in the colors of the *G* program. The reduction was made in this way for two reasons: (1) the observations of the K stars were more than twice as numerous as those of the G stars, and, consequently, the colors of the former in different regions of the sky are more accurately connected with one another, and (2) a comparison of the colors of the K stars with other data, particularly with the colors of Becker,[4] shows no definite systematic trend with right ascension.

REDUCTION OF MAGNITUDES TO SYSTEM OF K PROGRAM

Since the photoelectric response of the photometer was found to be linear with light intensity, no scale factor was used in the reduction of the magnitudes. A comparison of the observed magnitudes with those found for the same stars on other nights indicated that some change had taken place during the course of the observations on seven of the thirty-five nights. The observations on these nights were separated chronologically into two parts, and two additive constants were used instead of one in reducing the magnitude observations of these seven nights to a homogeneous system.

One constant was used to reduce the zero point of the magnitudes of the G stars to that of the K stars. This zero point is believed to correspond closely with that established by the precepts of the International Astronomical Union. No scale difference between the magnitudes of the stars common to the two programs is evident.

The various columns in Table 1 contain the following data: (1) and (2) the BS and the Boss numbers, respectively; (3) the color observed when the *G* program was in progress, reduced to the *K* system in the manner explained; (4) the color from the *K* program; (5) the difference (3) *minus* (4) in units of 0.001 mag.; (6) the weighted mean color; (7) and (8) the number of nights on which each star was observed with reference to the *G* and *K* programs, respectively; (9), (10), (11), and (12) the corresponding data for the magnitudes. The

[4] *Veröff. Sternwarte Berlin Babelsberg*, **10**, No. 3, 1933.

large discordance in the magnitudes found for Boss 1806, 42 ω Gem., tends to confirm Fath's suspicion[5] that this star is a variable.

TABLE 1

COMPARISON OF THE OBSERVATIONS

STAR		COLOR			MEAN	NO.		MAG.			MEAN
BS	PGC	K'	K		S_c	$n_{K'}$	n_K	K'	K	Δm	
351	270	+0^{m}023	+0^{m}025	− 2	+0^{m}024	10	9	3^{m}52	3^{m}54	−0^{m}02	3^{m}529
509	391	− .031	− .060	+29	− .043	3	2	2.52	2.56	− .04	2.536
824	634	+ .051	+ .030	+21	+ .043	3	2	3.34	3.36	− .02	3.348
882	670	+ .123	+ .104	+19	+ .119	2	8	3.58	3.61	− .03	3.604
991	746	+ .183	+ .192	− 9	+ .188	3	4	3.27	3.28	− .01	3.276
1015	770	+ .100	+ .110	−10	+ .104	6	4	3.74	3.73	+ .01	3.736
1256	936	+ .027	+ .035	− 8	+ .033	2	6	3.20	3.22	− .02	3.215
1684	1234	+ .252	+ .221	+31	+ .226	1	5	3.42	3.37	+ .05	3.378
1739	1266	− .001	+ .009	−10	+ .007	2	6	3.92	3.85	+ .07	3.868
1805	1311	+ .157	+ .156	+ 1	+ .156	6	6	3.60	3.56	+ .04	3.580
2480	1722	+ .179	+ .191	−12	+ .187	4	8	3.80	3.79	+ .01	3.793
2630	1806	+ .008	+ .009	− 1	+ .009	2	6	4.20	4.08	.....	Var.
3306	2234	− .007	− .005	− 2	− .005	1	5	4.13	4.08	+ .05	4.088
3366	2271	+ .093	+ .090	+ 3	+ .091	3	7	4.00	3.96	+ .04	3.972
3731	2524	+ .089	+ .090	− 1	+ .090	3	5	3.14	3.10	+ .04	3.115
3815	2573	− .072	− .059	−13	− .062	1	3	4.47	4.47	.00	4.470
4258	2910	+ .069	+ .085	−16	+ .077	5	4	3.81	3.78	+ .03	3.797
4667	3194	+ .016	+ .009	+ 7	+ .013	6	4	3.89	3.83	+ .06	3.866
4924	3379	+ .095	+ .084	+11	+ .091	7	4	3.63	3.63	.00	3.630
5201	3573	+ .196	+ .178	+18	+ .183	2	5	3.37	3.32	+ .05	3.334
5616	3842	+ .103	+ .102	+ 1	+ .102	4	7	3.13	3.12	+ .01	3.124
6103	4169	+ .035	+ .003	+32	+ .006	1	9	3.80	3.78	+ .02	3.782
6270	4294	+ .086	+ .091	− 5	+ .090	3	8	3.73	3.77	− .04	3.759
7181	4826	+ .117	+ .126	− 9	+ .125	1	9	3.91	3.86	+ .05	3.865
7478	5021	+ .003	+ .026	−23	+ .023	1	8	3.56	3.61	− .05	3.604
7956	5345	+ .142	+ .135	+ 7	+ .137	5	11	3.49	3.55	− .06	3.531
8475	5724	+ .073	+ .086	−13	+ .081	6	11	4.08	4.15	− .07	4.125
8796	5954	+0.154	+0.162	− 8	+0.159	4	7	3.28	3.33	− .05	3.312

It is evident that the comparison of the magnitudes K and K' shows systematic differences amounting to several hundredths of a magnitude, which depend on the right ascension. It does not seem desirable to correct either system of magnitudes until a third system of comparable or greater accuracy has been determined in this

[5] *Pub. A.S.P.*, **43**, 292, 1931.

region of the spectrum. Since the magnitude observations of the G program are given in detail in Table 2, which is similar to that already published for the K stars, it will be possible to adjust these magnitudes systematically at any future date.

ACCURACY

If we assign equal weight to systems K and K', the probable error of the mean color, K or K', of one star in Table 1 is 0.007 mag. The corresponding figure for those stars observed at least three times in each program is 0.005 mag. The probable error of the mean magnitude, K or K', of one star is 0.020 mag. Had two constants been used in place of one in the reduction of the zero points, as was done in the case of the colors, the probable error of a single star on either system would be reduced to 0.013 mag. These infrared magnitudes are used in conjunction with photographic magnitudes to form colors. The scale of this system of colors is approximately six times that of the Sproul colors (BG17 − RG9); consequently, a systematic error of 0.03 mag. in either system of magnitudes would cause an error of 0.005 mag. in the reduced colors. The probable error of one observation of color on one night, derived from the internal agreement of the observations of the G program, is 0.010 mag. The probable error of one magnitude is 0.024 mag.

COMBINATION WITH OTHER DATA

For the sake of brevity, the colors observed in the G program reduced to the system previously published for the K stars and those of the K program will be denoted as S_c. Since the colors derived from the combination of the infrared and the photographic magnitudes and those observed in the shorter wave lengths by Becker are closely related to S_c, these two systems were reduced to S_c in order to obtain a mean color of greater reliability than that of its constituents.

Becker's colors for all stars in common with S_c for which the accurate spectral classification is earlier than K1 are plotted against S_c in Figure 2. The straight line represents the relationship

$$S_b = 0.561B - 0.182 \text{ mag.},$$

TABLE 2*

MAGNITUDE RESIDUALS

Star PGC	'36 Sep 25	'36 Oct 11	'36 Oct 19	'36 Oct 20	'36 Oct 24	'36 Oct 26	'36 Nov 5	'37 Jan 4	'37 Jan 9	'36 Jan 12	'37 Dec 31	'37 Nov 3	'37 Nov 20	'37 Nov 21	'37 Jan 13	'36 Jan 23	'36 Jan 20	'36 Jan 24	'36 Feb 18	'36 Feb 19	'36 Feb 22	'36 Jan 23
41	-	-	-	0	-	-	-1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
104	-1	0	-	-	-	-	-	-	-	+2	+1	0	-	-	-	-	0	-	-	-	-	-
125	-	-	-	+2	-	-	+3	-	-	-4	-	-	-	-	-	-	-	-	-	-	-	-
130	+1	-2	-	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
138	-	-	-	0	-	-	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
162	-8	0	-	-	-	-	-	-	-	-	-	-2	-	-	+2	-	+6	-	-	-	-	-
171	-7	0	-	-	-	-	+7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
244	-	-	-	-	-	-4	-	-	-	-	-	+3	-	-	-	-	0	-	-	-	-	-
261	-	-	-	-	+5	-	-	-	-	-	-	-1	-	-	-	-	+5	-	-	-	-	-
262	-	-	-	-	-	+1	-	-	-	-	+1	+3	-	-	-	-	+4	-	-	-	-	-
270	+10	-	-	-2	+3	-	-	-	-1	0	-	-8	-	-4	+5	+4	-1	-	-	-	-	-
330	-	-	-	-	-	-3	-	-	-	-	-	+2	-	-	-	+1	-	-	-	-	-	-
335	-	-	-	-	+1	-	-	-	-	-	-	-15	-	-	+2	-4	-	-	-	-	-	-
350	-	-	-	-	-	-1	-	-	-	-	-	-1	-	-	-	+3	-	-	-	-	-	-
372	-	-	-	-	-	-1	-	-	-	-	-	-1	-	-	-	+2	-	-	-	-	-	-
383	-	-	-	-	+3	-	-	-	-	-	-	-	-	-	+1	-3	-	-	-	-	-	-
391	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-1	+3	-2	-	-	-	-
439	-	-	-	-	-	+1	-	-	-	-	-2	+2	-	-	-	-2	-	-	-	-	-	-
450	-	-	-	-	+3	-	-	-	-	-	-4	-	-	-	-2	+2	-	-	-	-	-	-
495	-	-	-	+10	-	-	-	-	-	-	-	-5	-	-	-5	-5	-	-	-	-	-	-
542	-	-	0	-	-	-	-	-	+1	0	-	-	-	-	-3	-	-	-	-	-	-	-
561	-	-	+5	-	-	-	-	-	-	-	-	-1	-	-	-	-6	-	+2	-	-	-	-
585	-	-	-	0	-	-	-	-	-	-	-	-	-	-	-	-3	-	+3	-	-	-	-
619	-	-	-7	-	-	-	-	-	+3	+2	-	-	-	-	+2	-	-	-	-	-	-	-
634	-	-	-	-	-	-	-	-	-	-	-	-2	-	-	-	+5	-3	-	-	-	-	-
670	-	-	-	-	-	-	-	-	-	-	-	-4	-	-	-	-	+5	-	-	-	-	-
672	-	-	+3	-	-	-	-	-	-1	-	-	-	-	-	+2	-	-3	-	-	-	-	-
704	-	-1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	+6	-	-6	-	-	-
710	-	-	+3	-	-	-	0	-	-	-	-	-	-	-	+2	-	0	-	-	-	-	-
746	-	-	-	-	-	-	-	-	-1	-	-	-	-	-	0	+1	-	-	-	-	-	-
751	-	+6	-	-	-	-	-	-	-	-	-	-3	-5	-	+1	-	+2	-	-	-	-	-
752	-	-1	-	-	-	-	-	-	-	-	-	-5	+4	-	+2	-	0	-	-	-	-	-
763	-	-1	-	-	-	-	-	-	-	-	-	-	-	-	-	-1	+3	-	-	-	-	-
770	-4	-1	0	-	-1	-	+2	-	-	-	-	-	-	+3	-	-	-	-	-	-	-	-
825	-	+3	-	-	-	-	-	-	-	-	-	-6	+4	-	+3	-	-3	-	-	-	-	-
835	-	+4	-	-	-	-	-	-	-	-	-	-	-	-	-4	-	0	-	-	-	-	-
876	-	-	+5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-6	+2	-	-	-
916	-	-	-	-	-	-	-	-	-	-	-	-	-	+1	-	-3	-	-	-	-	-	-
952	-	-	-	-	+4	-	-	-	-	-	-	-	-	-	-	-6	-	-	+2	-	-	-
988	-	-	-	-	-	+2	-	-	-	-	-	-	-	-	-	-	-7	+6	-	-	-	-
999	-	-	-	-	-	0	-	-	-	-	-	-	-	-	-	+1	0	-	-	-	-	-
1008	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-4	-	-	+4	-	-	-
1042	-	-	-	-	-	0	-	-	-	-	-	-	-	-	-	-2	-	-	+3	-	-	-
1134	-	-	-	-	-	-5	-	-	-	-	+2	-	-	-	-	-	0	+1	-	-	-	-
1185	-	-	-	-	-	+1	-	-	-	-	-	-	-	-	-	-	+2	-4	-	-	-	-
1234	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	-	-	-	-	-	-	-
1259	-	-	-	-	-	+1	-	-	-	-	-	-1	-	+1	-	-	-2	-	0	-	-	-
1266	-	-	-	-	-	-	-	-1	-	-	-	-	-	-	+1	-	-	-	-	-	-	-
1294	-	+1	-	-	-	+1	-	-	-	-	-	+1	-	+2	-	-	-1	-	-1	-	-	-
1311	-	+3	-	-	-	+1	+1	-1	-	-	-	-	-	-	-1	-2	-	-	-	-	-	-
1407	-	-2	-	-	-	+4	-	-	-	-	-	-	-	-	0	-3	-	-	-	-	-	-
1441	-	-3	-	-	-	+3	-	-	-	-	-	-	-	-	+1	-2	-	-	-	-	-	-
1582	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	-	0	-	-	-	-
1627	-	-	-	-	-	-1	-1	-	-	-	-	+3	-	-	-	-	-	-	+1	-	-	-
1628	-	-	-	-	-	-	-	-	-	-	-	-	+3	-	-1	-	-1	-	-	-	-	-
1694	-	-	-	-	-	-	-	+1	-	-	-	-2	-	-	-	-	-	-	0	+3	-	-
1720	0	-	-	-	-	0	-5	-	-	-	-	+5	-	-	-	-	-	-	+1	-	-	-
1722	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-5	+1	+8	-	-	-3	-	-
1724	-	-	-	-	-	-	-	-1	-	-	-	+2	-	-	-	-	-	-	+1	0	-	-
1780	-	-	-	-	-	-	-	-1	-	-	-	-	-	-	-	-	0	-	-	+1	-	-
1806	-	-	-	-	-	-2	-	-	-	-	-	-	-	-	-	+2	-	-	-	-	-	-
1822	-	-	-	-	-	-	0	-	-	-	-	-0	-	-	-	-	-2	-	0	-	-	-
1866	-	-	-	-	-	-	0	-	-	-	-	0	-5	-	-	-	-	+4	-	-	-	-
1865	-	-	-	-	-	-	-	-	-	-	9	-	0	+1	-1	-	-	-	-	-	-	-
1921	-	-	-	-	-	-	-	-	-	-	-1	-	-	-	+1	-	+1	-	-	-	-	-
1936	-	-	-	-	-	-	-	-	-	-	-	-	-1	-1	+2	-	-	-	-	-	-	-
1957	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	-	-	-	-	-	-
2006	-	-	-	-	-	-	-	+1	-	-	-	-	+1	-4	-	-	-	-	-	-	-	-
2146	-	-	-	-	-	-	-	-	-	-	-	+2	-	-	+2	-	-3	-	-	-	-	-
2150	-	-	-	-	-	0	-	+2	-	-	-	-	-	-3	-	-	-	-	-	-	-	-
2157	-	-	-	-	-	-	-	-	-	-	-	-	0	-	+2	-	-1	-	-	-	-	-
2174	-	-	-	-	-	0	-	-2	-	-	-	-	-	+1	-	-	-	-	-	-	-	-
2205	-	-	-	-	-	-	-	-	-	-	-	-	-	-	+1	-	-2	-	-	-	-	-
2231	-	-	-	-	-	+2	-	-	-	-	-	-	-	-2	-	-	-	-	-	-	-	-
2234	-	-	-	-	-	-	-	-	-	-	-	0	-	-	-	-	-	-	-	-	-	-
2247	-	-3	-	-	-	-	-	-	-	-	-	-	-	+2	-	-	-	-	-	-	-	-
2271	-	-	-	-	-	-	-	-	-	-	-	-	-	0	-4	-	-	+4	-	-	-	-
2284	-	-	-	-	-	-	-	-	-	-	-	-	-	0	-	-	-	-	-1	-	-	-
2289	-	-	-	-	-	-1	-	-	-	-	-	-	+2	0	-	-	-	-	-	-	-	-
2368	-	-	-	-	-	-	-1	-	-	-	-	-3	-	-	-	-	-	-	+3	-1	-	-
2391	-	-	-	-	-	-	-	-	-	-	-3	-	-	-	-	-	-1	+3	-	-	-	-
2409	-	-	-	-	-	-	-	-	-	-	+2	-	-	-	-	-	0	-2	-	-	-	-
2437	-	-	-	-	-	-	-3	-	-	-	-	-	-	-	-	-	-	+2	-	-	-	-
2444	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-3	+2	-	-	-	-
2478	-	-	-	-	-	-	-	-	-	-	0	-	-	-	-	-	-2	+1	-	-	-	-
2524	-	-	-	-	-	-	-2	-	-	-	-	-	-	-	-	-	+1	-	+2	-	-	-
2530	-	-	-	-	-	-	+3	-	-	-	-2	+1	-	-	-	-	-	-	-	0	-	-
2549	-	-	-	-	0	-	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2555	-	-	-	-	-	-	0	-	-	-	-	-	-	-	-	-2	-	+1	-	-	-	-
2566	-	-	-	-	-	-	0	-	-	-	-	-	-	-	-	-	-	-	+2	-	-	-
2573	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	-	-
2626	-	-	-	-	-	-	+3	+3	-	-	-	-	-	-	-	-	-	-2	-	-3	-	-
2660	-	-	-	-	-	-	-6	-	-	-	-	-	-	-	-	-	-	-	+6	-	-	-
2681	-	-	-	-	-	-	+1	-	-	-	-	-	-	-	-2	-	+1	-	-	-	-	-
2829	-	-	-	-	-	-	-	-	-	-	+1	-	-	+1	-	-	-1	-	-	-	-	-
2890	-	-	-	-	-	-	-	-	-	-	-	-1	-	+1	-	-	-	-	0	-	-	-
2618	-	-	-	-	-	-	0	-	-	-	-	-	-	-	-	0	-1	-	-	-	-	-

Star PGC	'37 Apr 30	'37 May 1	'37 Jun 8	'37 Oct 15	'37 Dec 9	'37 Jan 4	'37 Jan 21	'36 Jan 13	'36 Jan 24	'36 Apr 24	'36 Jan 24	'36 Feb 18	'36 Feb 19	'36 Feb 22	'36 Apr 26	'36 May 14	'36 May 23	'36 May 28
2910	-	-	-	-	-	0	-	+4	-	-	+4	+1	-7	-	-	-	-	-
2920	-	-	-	-	-2	-	-	-2	-	-	+5	-	-	-	-	-	-	-
2950	+3	-	-	-	-	-	-	-	-	0	-	-4	-	-	-	-	-	-
2995	-	-	-	-	-	-	-	-	-	-	+3	-3	-	-	-	-	-	-
3000	-	-	-	-	-	-	-	-	-	+4	-	-3	-	-	-	-	-	-
3007	-	-	-	-	0	-	-	-	-	-	-	-	-1	0	-	-	-	-
3046	-	-	-	-	-	-	-	-	-	-	-	-	+1	0	-	-	-	-
3074	-	-1	-	-	-	-	+2	-	-	-3	-	+2	-	-	-	-	-	-
3075	-	-	+2	-	-	-2	-3	-	-	-	-1	+3	-	-	-	-	-	-
3155	-3	-	-1	-	-	-	-	-	-	-	+3	+1	-	-	-	-	-	-
3181	-1	+2	-	-	-	-	-	-	+1	-1	-1	-	-	-	-	-	-	-
3279	-	-5	-	-	-	-	-	-	+4	-	+3	-	-	-	-	-	-	-
3294	-	-	-	-	-	-	-	+1	+1	-	-	+2	-3	-1	+2	-	-	-
3347	+3	-	-	-	-	-	-2	-	-2	+3	-1	-	-	-2	-	-	-	-
3379	-	-1	-	-	-	+2	-	-	+4	-	-	+2	0	-2	-7	-	-	-
3381	-	+1	0	-	-	-	-	-	-	0	-3	-	-	-	-	-	-	-
3424	0	-	-2	-	-	-	+2	-	-	-	+1	-2	-	-	-	-	-	-
3443	+3	-	-2	-	-	-	-	-	-	+5	-7	-	-	-	-	-	-	-
3487	-	-	-1	-	0	-	-	-	+3	-	-3	-	-	-	-	-	-	-
3497	-	-	-	-	+1	-	-	-	-1	-	-	-	-	-	-	-	-	-
3533	+3	-	-	-	-	-	-	-	-1	-	0	-	-	-	-	-	-	-1
3554	+6	-	-	-	-	-	-	-	-	-	-	-	-4	-1	-	-	-	-
3573	-	-	-	-	-	-	-	-	-	-	-1	-	-	-	+1	-	-	-
3655	-	+2	-2	-	-	-	-	+3	-	-4	-	-	-	-	-	-	-	-
3715	-	-	-	-	-	-	-	-	-	-	-	-	-	+9	-6	-2	-	-
3753	-8	-	-	-	-	-	-	+5	-	-	-	-	-	+6	-	-	-	-
3754	+3	-	-	-	-	-	-	-1	-	-	-	-	0	-1	-	-	-	-
3798	+1	-	-	-	-	-	-	0	-	-	-	-	-	-1	-	-	-	-
3842	-	-	-	-	-	-	-	-3	-	-	-	-	-	-2	+5	-1	-	-
3856	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-2	+2	-
3893	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	-1	-
3894	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0	-	-	-
3895	-1	-	-	-	-	-	-	-	-	-	-	-	-	-0	0	-	-	-
3958	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0	-
3979	-1	-	-	-	-	-	-	-	-	-	-	-	-	+1	+1	-	-	-
3992	-	-	-	-	-	'37 Nov	'36 Sep 25	'36 Sep 26	'36 Oct 11	'36 Oct 12	'36 Oct 13	'36 Oct 20	'36 Oct 24	'36 Oct 25	-	-2	-	+1
4000	-	-	-	-	3									-	0	-	0	0
4010	-3	-	-	-	-	-	-	-	-	-	-	-	-	+1	+2	-	-	-
4024	-	-	-	-	-	-	-	-	-	-	-	-	-	-1	0	-	-	-
4040	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	-	0	-
4042	+5	-	-	-	-	-	0	-	-	-	-	-	-	0	-2	0	-	-
4075	-2	-	-	-	-	-	-	-	-	-	-	-	-	-1	-	+2	-	-
4160	-	-	-	-	-	0	-	-	-	-	-	-	-	-	-	-	-	-
4192	-	-	-	-	-	-1	0	-	-	-	-	-	-	-	-	0	-	-
4207	-1	-	-	-	-	-	-	+3	-	-	-	-	-	-2	-	+2	-	-
4228	-	-	0	-	-	0	0	-	-	-	-	-	-	-	-	-	-	+1
4240	0	-	+4	-	-	-2	-3	-	-	-	-	-	-	-	-	-	-	-
4245	+21	-	+1	-	-	-3	-	-	+1	-	-	-	-	-	-	-	-	-
4294	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	-	+1	0
2279	-	-	-	-	-	-	0	-	-3	-	-	-	-	-	-	-	-	+3
4403	-	-	+2	-	-	-	-	-	-1	-1	-	-	-	-	-	-	+1	-
4443	-	-	-	-	-	-	+3	-	+2	-	-	-	-	-4	-	-	-	-
4488	-4	-	0	-	-	-	-	-	+1	+5	-	-	-	-	Oct	Oct	Nov	-1
4497	-	-	-	-	-	-	-	-	+5	0	-	-	-	26	28	5	-5	
4580	-	-	-8	-	-	+5	-	-	0	-	-	-	-	-	-	-	+2	
4589	-2	-4	-	-	-	-	-	-	+3	-	-	-	-	+1	+2	-	-	
4601	-	-	-	-	-	-	-	-	-	-	-	-	-	+1	0	-	-	
4635	-	-	-	-	-	+1	+3	-	-	-	-	-	-	-	-	-	+3	
4782	-3	-	-	-	-	-	-	-	+1	-	-	+2	-	0	-	-	-	
4787	0	+1	+6	-	-	-	-	-	-6	-	-	0	-1	-	-	-	-	
4799	+3	-	-	-	-	-	-	-	0	-	-	-1	0	-	-	-	-	
4815	-	0	-	-	-	+3	-	-	+4	-	-	-	+1	-	-	-	-	
4826	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0
4848	+6	-	-	-	-	-	-	-	-4	-	-	-1	-1	-	-	-	-	
2495	-	-	-	-	-	-	-	-	0	-	0	-	-	-	-	-	-	
4892	-	-1	-	-	-	-	-	-	-4	-	-	-	+5	-1	-	-	-	
4898	-3	-	-	+3	-	+8	-	-	-	-	-6	-2	-	-	-	-	-	
4950	0	-	-	-	-	-	-	-5	+1	+2	-	-	-	-	-	-	-	
4958	-	-	-	-	-	-	-	-	0	-	-	+2	-1	-	-	-	-	
5012	-	-	-	-	-	-	-	-1	-	-	-	-1	+1	-	-	-	-	
5021	-	-	-0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
5023	-2	-	+4	-	-	-	-	-	-	-	-2	-1	-	-	-	-	-	
5039	-	-	-	-	-	-	-	-	+1	-3	-	-	-	-	-	-	+2	
5065	-	-	+3	-	-	-	-	-	-5	-]	+1	-	-	+1	-	-	-	
5096	-	'37 Dec 8	-	-	-	-	-	-	+5	-	-	+1	-6	-	-	-	-	
5146	-	-	-	-	-	-	-	-	+2	+2	-4	-	-	-	-	-	-	
5192	-	+3	0	+7	-	-	-5	-	-	-	-	-	-	-	-	-	-	-4
5193	-	-	-	-	-	-	0	-	-	-	-1	-	-	-	-	-	-	
5197	-	-	-	-	-	-	-	-	-5	-	-	-	-	-	+2	+4	-	
5229	-	-	-	-	-	-	-	-	-3	+4	-	-	-	-	-	0	+3	
5304	-	-	0	+5	-	-	-	-	-2	-2	-	-	-	-	-1	-2	-	
5344	-	-	+5	-	-	-	+1	-	-	-	0	-3	-	-2	-			
5345	-	-	-2	-	+2	-	+5	-4	-1	-	-	-	-	-	-	-	-	
5373	-	-	-2	+1	-	+1	-	-	-	-	-	-	-	+5	-	-4		
5527	-	-	-2	-	-	-	-	-	+3	-	-	-	-	-4	+2	-		
5590	-	-	-1	+3	-	-	-	-	-1	-	-	+2	-1	-	-	-		
5724	-	-	+2	+5	-	+3	-	-	-	-4	-5	-	+2	-	-	-		
5786	-	-	0	-	-	-	-	-	+3	-	-	-1	-	-	-	-		
5840	-	-	-	-4	-	-	+5	-	-	-	-	-	-	-1	-	-		
5851	-	+2	-	+2	-	-	-	-	+2	-	-	-2	-	-4	-	-		
5947	'38 Jan 12	-	-	-	-	-	-	-	0	-	-	-	-	0	-	-		
5931	-	-	-	-	-	-	0	-	-	-6	-	+5	-	-	-	-		
5954	-1	-	-	-	+2	-	0	-	-	-	-	-2	-	-	-	-		
5959	-	-3	+2	+4	-	-	-	-9	-	-	-	+7	-	-	-	-		
6024	-	-	-5	+3	-	-	-	+6	-	-	-3	-	-	-	-	-		
6037	-	-	-	+6	-	-	-	-6	-	-	-	0	-	-	-	-		
6172	-	-	-	+4	-	-	-	+1	-	-	-	-	-	-	-	-		

* The infrared magnitudes used as a basis for these residuals are shown in column 5 of Table 8. The unit is 0.01 mag. A positive sign means that the star was observed brighter than average. These observations were made in 1936 and 1937 on the dates indicated at the top of each column. If a star was observed after midnight, the residual for this observation is listed under the preceding date. A heavy line separates the observations of the seven nights when two constants were used to reduce the observed magnitudes to a common zero point. Several very large residuals are included above but were given no weight in forming the mean magnitudes of Table 8.

which I have adopted in reducing Becker's color B to the Sproul system S_c.

The difference in magnitude between RG9 and the photographic magnitudes by Hertzsprung[6] was used as a third system of colors.

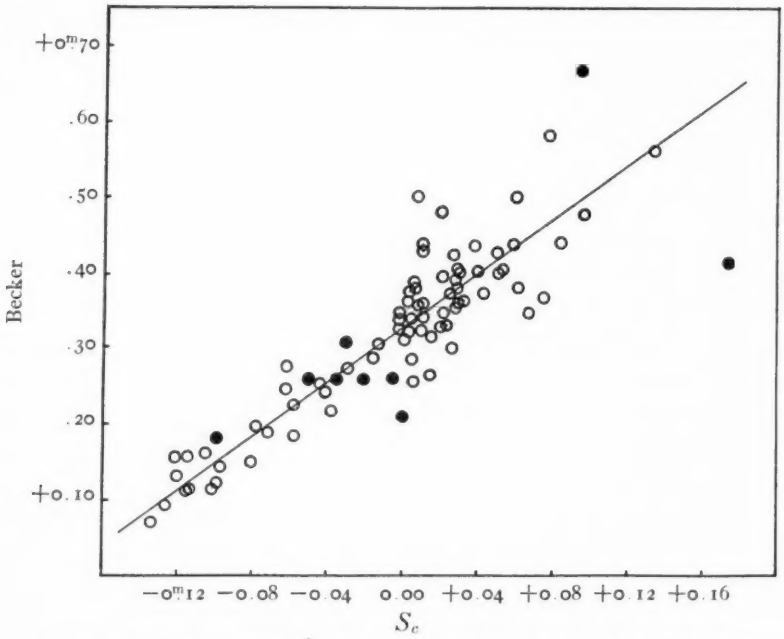


FIG. 2.—The ordinate is Becker's blue-violet color and the abscissa is the Sproul red-infrared color. The filled circles are c-stars. The c-star at the top of the diagram, ϵ Gem, has not been observed at Sproul, but is included since it is a star of special interest. Its Yale infrared color, reduced to the Sproul scale, has been used. The c-star at the right, Boss 5931, is one of extremely high luminosity. The observations of many of the more discordant stars in this diagram are of relatively low weight.

The photographic magnitudes of Göttingen Actinometry B[7] reduced to the Hertzsprung scale were also used whenever available. The reduction to the Sproul system S_c was made by means of the relationship

$$S_m = 0.164(\text{Phtg.} - \text{RG9}) - 0.343 \text{ mag}.$$

[6] *B.A.N.*, **1**, No. 35, 1923.

[7] *Abhandlungen der Gesellschaft der Wissenschaften zu Göttingen, Math.-phys. Klasse*, **8**, No. 4, 1912.

Since both S_c and S_m contain RG9, they are not entirely independent; nevertheless, it is instructive to compare the relative accuracy of the three sets of colors. A comparison of this sort involving 57 stars common to the three indicated that the probable error of each was 0.010 mag. Despite the objection just mentioned, this result indicates that the three sets of colors are closely related to one another and that the cosmical errors are small or of the same order as the errors of observation. This statement is not quite true for the c stars, but these are few in number and unusually red for their spectral type regardless of the spectral region in which their colors are measured.

I have used as many of these three sets of colors as were available to form a mean color for each star in the subsequent discussion. In the case of close binary stars, these mean values were used if the difference in magnitude between the components was large. The mean color was also used for the 3 stars known to have components of very similar spectral type.

I am grateful to Miss Hoffleit, of the Harvard Observatory, for sending me accurate spectral classifications of 37 stars observed in the *G* program. Since the Mount Wilson,[1] Victoria,[8] and these Harvard classifications agree very well with one another for G stars, they are combined whenever possible to form a mean spectral type for each star.

DISCUSSION

The main purpose of this program is to investigate the degree of accuracy with which the absolute magnitudes of G-type stars may be determined from accurate colors and spectral types. It was shown in the former paper[2] that absolute magnitudes derived from color excess agree well with trigonometric absolute magnitudes and with the Victoria spectroscopic absolute magnitudes for K-type giants. For the G stars in that program the agreement with the Victoria results was good—but it was poor with the trigonometric data. Unfortunately, the trigonometric absolute magnitudes of G-type giants are uncertain; consequently, other methods of obtaining their absolute magnitudes have been used. The following discussion includes stars of both programs whose accurate mean spectral type ranges

[8] *Pub. Dom. Ap. Obs.*, **3**, 48, 1927.

from G2.0 to K0.3. The mean colors of both programs were formed in the same manner as that outlined in the previous section.

The stars were first divided into groups of the same spectral subtype. The color of each star was then plotted against its Victoria absolute magnitude. The color corresponding to absolute magnitude 2.1 in each subtype was then read from each graph. This color for each subtype is shown in the first two columns of Table 3. The difference was then taken between the observed color of each star and the color it would have if its absolute magnitude were 2.1. This difference is the color excess used in Table 3 to define a relationship between color excess and absolute magnitude.

TABLE 3

Mean Type	Color	C.E.	M_c
K0.0	$+0^m032$	$+0^m15$	-2^M0
G9.0	+ .017	+ .10	−1.1
G8.0	+ .012	+ .08	−0.6
G7.0	− .007	+ .06	−0.1
G6.0	− .014	+ .04	+0.5
G5.0	− .030	+ .02	+1.2
G4.0	− .030	.00	+2.1
G3.0	− .050	− .02	+3.2
G2.0	−0.070	− .04	+4.3
		− .06	+5.4
		− .08	+6.5

For the giant stars this relationship is based primarily on the Victoria absolute magnitudes, M_v. Some consideration was also given to mean absolute magnitudes computed from the τ and v components of the proper motion and to the absolute magnitudes of two stars in the Taurus Cluster. In deriving the absolute magnitudes from the τ and v components, the stars were separated into groups on the basis of the Victoria data. When the Mount Wilson data are used for the same purpose, there is very little change in the color excess of giants with absolute magnitude. The trigonometric absolute magnitudes were used for the dwarfs. Since there is some reason to believe, on the basis of the data presented in the final section, that the colors of some of these dwarfs may be influenced by red companions, some allowance for this suspicion has been made in

the construction of Table 3. The agreement of the absolute magnitudes computed from the color excess, M_c, with other data is shown in Figures 3, 4, and 5.

Figures 4 and 5 show that the agreement between M_c and M_v is much better than between M_c and M_w. The average difference between M_c and M_v for 87 stars with at least two of the three systems of color is 0.61 mag., and for 37 stars for which only S_c exists this difference is 0.77 mag. The graphs show that the agreement is better

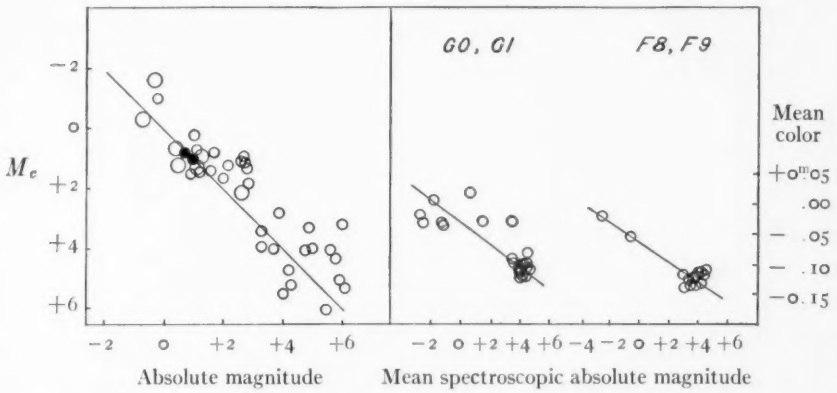


FIG. 3.—The larger circles in the diagram at the left represent groups of stars whose mean absolute magnitude was computed from the τ and v components of their proper motion. From left to right the number of stars included in each large circle is 5, 7, 13, 20, 29, 32, and 8, respectively. The abscissae of the small circles in the same diagram were computed from trigonometric parallaxes when the probable error of the parallax given by Schlesinger is less than one-fifth its value. The two filled circles represent two stars in the Taurus cluster.

for the giants and poorer for the dwarfs than these differences indicate. If the Victoria results represent the true frequency distribution of the absolute magnitudes of these stars, the method of determining absolute magnitude from color excess is comparable in accuracy to the spectroscopic method. If the Mount Wilson frequency distribution is correct, this method is useful only for statistical purposes in distinguishing giants from dwarfs or from supergiants. Stenquist[9] has interpreted his results in the same way from a discussion of absolute magnitudes derived from a cyanogen criterion. He suggests

[9] *Medd. från Astronomiska Obs. Uppsala*, No. 72, 46, 1937.

that the extremely small dispersion within the Mount Wilson giant groups is "probably an effect of a relative insensibility of the luminosity criteria used for these stars." Gratton[10] and Gyllenberg[11] have derived frequency distributions for giant HD K0–K2 stars which resemble the Victoria rather than the Mount Wilson distributions. The marked differences in these distributions are clearly brought out in a paper by W. W. Morgan.[12]

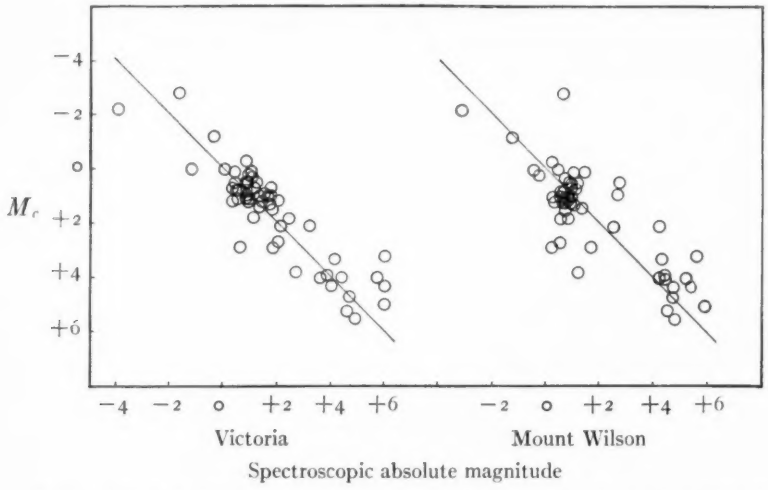


FIG. 4.—Each diagram includes stars whose mean spectral types range from G2 to K0. These stars are those observed in connection with the G-program and included in the Victoria list of absolute magnitudes.

The method of deriving mean color excess and its relation to absolute magnitude explained in this paper differs slightly from that used in the former paper. It does not seem necessary to publish the revised absolute magnitudes of the stars in that paper which have been included in this discussion. The former may be reduced to the present system with reasonable accuracy by means of Table 4.

The colors of G0 and G1 stars as well as those of subtypes F8 and F9 are plotted against absolute magnitude at the right-hand side of Figure 3. There is very little cosmic dispersion in the colors of the dwarfs in either graph. The relationships denoted by the straight

[10] *B.A.N.*, No. 248, 1933.

[11] *Lund. Medd.*, Ser. 1, No. 145, 1936.

[12] *Ap. J.*, **85**, 380, 1937.

lines in this figure have been used to derive values of M_c for two dwarf stars included in the following section of this paper.

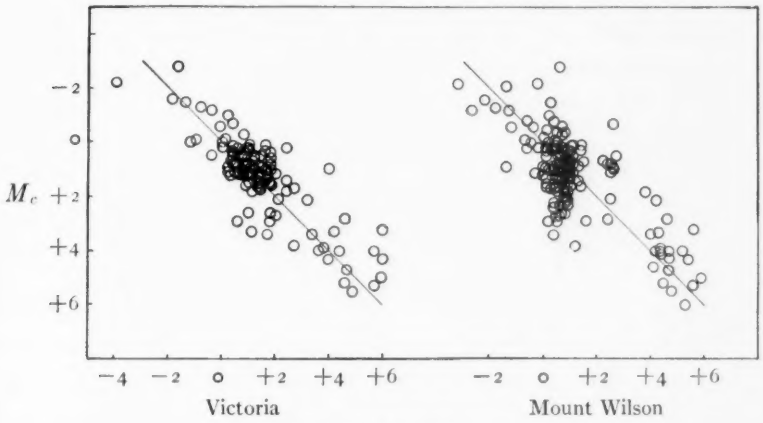


FIG. 5.—All G2 to K0 stars observed at Sproul are included in these plots. The difference in the frequency distribution of the absolute magnitudes of the Victoria and Mount Wilson catalogues is particularly noticeable for the giants.

No evidence of the possible influence of selective interstellar absorption is evident from a discussion of the relation of color excess of giant G stars with galactic latitude. Since spectroscopic absolute

TABLE 4

M_c

Formerly	Present
−2.4	−2.0
−0.9	−1.1
0.0	−0.4
+0.6	+0.2
+1.2	+0.9
+2.0	+2.1
+2.7	+3.3

magnitudes (but not parallaxes) should be comparatively free from this influence, the differences $M_c - M_v$ were plotted against galactic latitude. No correlation was evident. A change in color amounting to 0.10 mag. due to interstellar reddening would change the absolute magnitude derived from color excess by several magnitudes. Colors

of a few distant B stars have been observed at the Sproul Observatory[13] with the same filters as those used for the G stars. Several color excesses larger than 0.30 mag. were found. A comparison of colorimetric with spectroscopic absolute magnitudes of distant G stars of different galactic latitudes should shed additional light on the problem of interstellar scattering. Should a relationship between selective and general absorption be definitely established in a given region of the sky, the difference between the colorimetric and spectroscopic absolute magnitudes of G stars would enable the spectroscopist to reduce his absolute magnitude to a spectroscopic parallax.

CLOSE BINARY STARS

The influence of the fainter component on the composite color of close binary stars should be evident from a comparison of colors measured in widely separated regions of the spectrum. Also, since the absolute magnitudes of G stars may be derived from their color and spectral type, a discrepancy between this absolute magnitude and that determined by other methods may be attributed to the influence of the fainter component on the composite color.

The comparison of S_c with S_b should give a good indication of the color of the fainter component for those close visual binaries whose magnitude differences are known. When these differences are not known, the two criteria mentioned above should in certain cases enable us to decide whether or not the fainter companion is redder than the primary. For other stars these criteria should make it possible to discover the presence of dark companions. Such binaries might be too close for visual discovery and yet have periods of such length or orbits so inclined that it would be difficult to detect any variation of radial velocity.

The agreement between S_b and S_c is particularly good for F and dwarf G stars. Since the trigonometric absolute magnitudes of the dwarf G stars in the Sproul program are well determined, the criteria will be applied to this group. If these stars have unknown companions of early spectral type, such companions are probably white dwarfs and would not noticeably influence the observed colors.

The composite color of 10 known visual binaries whose trigono-

[13] *Ibid.*, **85**, 145, 1937.

metric absolute magnitudes are larger than +3.0 have been observed Data pertaining to these 10 stars are shown in Table 5.

In Table 5 the trigonometric absolute magnitude M_t represents that of the brighter component. According to Schlesinger's *General Catalogue of Stellar Parallaxes*, M_t is more uncertain for PGC 2950 and PGC 5851 than for the other stars, the parallax of the former being $0''.036 \pm 0''.006$ and of the latter $0''.032 \pm 0''.006$ (its two determinations are very discordant, $0''.024 \pm 0''.006$ and $0''.071 \pm 0''.014$). The values of M_c marked with an asterisk were computed from the color excess derived from S_c alone, while for the remaining stars as many of the three sets of colors as are available were used in forming the color excess. I am grateful to Dr. Kuiper[14] for the visual difference in magnitude between the components of six of the stars in Table 5. The differences shown for the other four stars were taken from other sources and are given to but one significant figure. The spectral type of the primary is given to tenths of a subclass when more than one accurate classification exists.

TABLE 5

CLOSE VISUAL BINARY STARS

Name	PGC	M_t	M_c	S_c	S_b	S_m	Vis Δm	Spectra
15 λ Aur	1259	4.0	4.3	$-0^m.114$	$-0^m.094$	$-0^m.106$	8.	G1.0
11 L Mi	2573	5.4	6.0	− .062	.027	.....	8.	G9.5
65 Leo	2950	3.4	0.5*	+ .033	.....	+ .021	4.33	G7
37 ξ Boo	3798	5.6	2.4*	− .021	.055	− .054	1.96	G6.0 K5
40 ζ Her	4246	3.3	2.6*	− .081	.098	− .096	2.80	G0.0
	4815	4.2	1.0*	− .079	.137	.....	1.78	F9.5
	4892	4.7	4.7	− .078	.....	.....	0.2	G4.5 G5
60 β Aql	5093	3.3	3.4	− .016	.021	− .036	7.	G7.3
	5851	3.8	1.9	− .046	.....	−0.047	0.55	G3 G3
85 Peg	6172	5.5	1.0*	−0.023	−0.092	.....	(3.5)	G3.0

Table 6 has been constructed in order to predict the amount of reddening to be expected from a companion of a given color temperature when a given difference in magnitude exists between the components. Star A represents the primary and is assumed to have a

[14] Unpublished.

color temperature of 6000°. Stars B and C are hypothetical companions with color temperatures of 4300° and 3300°, respectively. The change of color ΔS_c was computed with the aid of spectrophotometric grating observations shown in Figure 1 and of the known absorptions of filters RG9 and BG17 in the following manner: η Boo was assumed to have the color temperature of star A, α Ser that of B, and β UMi that of C. Consider first a binary AB where the visual difference in magnitude $B-A$ is 2.0 mag. The photoelectric response in all wave lengths for star B was arbitrarily decreased by a factor such that the visual response at 5500 A was 0.158 that of A. The corrected response for each wave-length band of B was then added to the observed response for the corresponding band of A.

TABLE 6

EFFECT OF DARK COMPANIONS ON COMPOSITE COLOR

	Δm	ΔS_c	ΔS_b	ΔS_m
$(A+B)-A$	2.0	$+0^m.044$	$+0^m.013$	$+0^m.020$
$(A+B)-A$	4.0	.015	.002	.004
$(A+C)-A$	2.0	.096	.022	.043
$(A+C)-A$	4.0	+0.018	+0.003	+0.008

These values $A+B$ were then multiplied by appropriate factors in order to give the observed energy distribution of such a composite star as a function of the wave length with each of the two filters in place. The ratio of the areas under the response curves for the two filters, expressed in magnitudes, was then determined, thus giving the composite color of two such stars when observed as one. The color of A alone was then determined by a similar process. The difference in color resulting from these two computations is shown under ΔS_c as 0.044 mag. in Table 6.

The corresponding change in Becker's colors, ΔS_b, was computed with the aid of Planck's law. All the energy through his filters was assumed to be concentrated at the effective wave lengths 4750 A and 4250 A. The effective wave lengths assumed for a similar computation of the change in color of S_m were 4250 A and 8300 A. Both ΔS_b and ΔS_m were multiplied by the scale factors mentioned in a previous section in order to reduce them to the scale of S_c. All

changes in color shown in Table 6 are necessarily positive, since the observed colors would always be increased by the addition of the faint red companion.

The disagreement in both the observed colors and the absolute magnitudes of Table 5 in the case of PGC 3798, 4815, and 6172 is so obvious and of such a nature as to indicate that the companion in each of these cases is considerably redder than the primary. The spectral type of the companion of PGC 3798 is known to be K5. Let us assume that its primary has a color temperature of 6000° and that its K5 companion, which is two magnitudes fainter, has a temperature of 4300°. When the colors S_c, S_b, and S_m are corrected for the influence of this companion by means of the data in Table 6, they are in good agreement with one another (−0.065, −0.068, and −0.074, respectively). The absolute magnitude of the primary computed from the mean of these three colors and the spectral type is then 5.1, a value which agrees reasonably well with the trigonometric absolute magnitude 5.6 given in Table 5. A comparison of Table 6 with Table 5 suggests that the colors of the companions of PGC 4815 and 6172 correspond to those of spectral types K5 and M, respectively.

M_t and M_c are in good agreement for PGC 1259, 2573, 4246, 4892, and 5093. The difference in magnitude between the components of three of these stars is probably greater than 5 mag. It seems probable that the color of the companion of 4246 is similar to that of the primary—it may be slightly redder, perhaps G5. The spectra of the components of 4892 are similar and the magnitude difference is small. The composite color is therefore the same as that of either component, and consequently no discordance in the absolute magnitudes is to be expected.

There are four other stars observed at Sproul whose absolute magnitudes computed from color and spectral type are noticeably less than those derived from well-determined trigonometric parallaxes. None of these stars is known to be a binary. The color observations of PGC 391 are discordant—this is undoubtedly partly due to the fact that this star is 16° south of the equator. Another, 5917, was classified as G5 at Victoria and as Go at Mount Wilson. If the Victoria classification is correct, no discrepancy exists. It seems prob-

able that the other two stars may be binaries with red companions. The evidence on which this suspicion is based is presented in Table 7.

Since the stars of Table 7 are near-by stars, it may be possible to confirm this suspicion by means of astrometric measures resulting from a search for stars with variable proper motions. The success or failure of the procedure just discussed relating to the discovery of faint companions by the two criteria used above rests on two assumptions. The first is that the colors S_c and S_b are closely related to each other for individual stars. The data discussed in this paper indicate that this is so for F and G main-sequence stars. The second assumption is that there is small cosmical dispersion in the relationship connecting absolute magnitude to color excess. Although one cannot be sure of this until all binary stars have been excluded from

TABLE 7

STARS SUSPECTED OF HAVING DARK COMPANIONS

Name	PGC	M_t	M_c	S_c	S_b	Sp.
30 M Cas....	244	5.8	3.6	-0^m058	-0^m080	G4.5
61 U Ma....	3075	5.6	3.4	−0.038	−0.060	G6.0

the data and the true frequency distribution of the absolute magnitudes of G stars has been definitely established, these preliminary results are encouraging.

Since this paper was written, a paper by W. W. Morgan[15] has appeared in which he finds that the colors of main-sequence stars of spectral types F5 to G0 show very little dispersion for a given sub-type. The data of this paper confirm this conclusion for F8 to G1 stars. They also make it more general, since this result holds equally well when spectral classifications other than the Yerkes classification are used and when red-infrared colors are substituted for those measured in the shorter wave lengths by Becker.

The various columns in Table 8 contain the following data: (1) the number of the star in the *Bright Star Catalogue* (same as HR); (2) the number of the star in the *Preliminary General Catalogue* by Boss; (3) the photographic magnitude on Hertzsprung's scale of

[15] *Ap. J.*, **87**, 460, 1938.

either Hertzsprung's value, that of Actinometry B, or the mean of both values; (4) the *Revised Harvard Photometry* visual magnitude; (5) the infrared magnitude on the international scale with filter RG9 in position; (6) the mean color observed at Swarthmore ($M_{BG17}-M_{RG9}$); (7) the number of nights on which the color was observed; (8) the mean color for as many of the three sets S_c, S_b, or S_m as are available, S_b is included in this mean whenever the mean color is followed by a colon, and S_m is included if there is an entry in column 3; (9) the mean spectral type (the spectra of those stars accurately classified in at least two observatories are tabulated to tenths of a subdivision); (10), (11), and (12) the absolute magnitudes derived at Victoria, Sproul (colorimetric), and Mount Wilson, respectively. The Sproul absolute magnitudes of the two known spectroscopic binaries included in the program are given in parentheses. The absolute magnitudes M_c for a few stars later than Ko were computed on the basis of the method cited in the previous paper dealing with this subject.[2] The last five stars are dwarfs which were observed primarily in connection with the K program. As no value of M_c was assigned to them in the former paper (since it discussed only giants), they are included in Table 8 (see pp. 340–43).

I wish to thank Mr. Roy W. Delaplaine for valuable assistance in reducing and checking the data discussed in this paper.

SPROUL OBSERVATORY
SWARTHMORE COLLEGE
SWARTHMORE, PENNSYLVANIA
June 1938

TABLE 8

CATALOGUE OF SWARTHMORE OBSERVATIONS

STAR		MAGNITUDES			COLOR	N	MEAN		ABSOLUTE MAGNITUDES		
BS	PGC	Ptg.	Vis.	Infr.	S_c		Color	Sp.	M_v	M_c	M_w
60	41	....	$5^m.80$	$4^m.66$	$+0^m.001$	3	.....	G4	.....	$+0^M.9$	$+0^M.0$
131	104	$6^m.49$	5.53	4.22	+ .045	6	$+0^m.037$	K0	.....	+1.8	+0.8
157	125	....	5.62	4.41	− .006	3	.....	G0	.....	.....	+0.6
163	130	5.35	4.52	3.29	+ .015	3	− .008:	G3.7	$+1^M.5$	+0.9	+2.6
175	138	....	5.42	4.27	+ .007	3	.....	G5	.....	+0.6	+0.9
213	162	6.58	5.68	4.44	+ .021	5	+ .015	G9	.....	+2.2	+0.7
222	171	6.68	5.82	4.66	+ .010	3	.000	K4.0	+7.0	.....	+6.5
321	244	....	5.26	4.21	− .058	4	− .069:	G4.5	+6.0	+4.3	+5.4
339	261	6.31	5.60	4.74	− .071	3	− .078	F5	.....	.....	+1.7
340	262	....	5.74	4.89	− .113	5	.....	F8.5	+2.6	.....	+3.6
430	330	....	5.33	4.17	+ .009	4	.....	G9.5	+0.6	+2.9	+0.2
437	335	4.73	3.72	2.53	+ .025	4	+ .026:	AG4.7	+0.0	+0.0	+0.4
458	350	4.72	4.18	3.39	− .145	4	− .134	F8.7	+4.0	.....	+3.7
483	372	....	5.10	4.16	− .097	4	− .099:	G1.0	+4.7	.....	+4.4
493	382	6.08	5.32	4.25	− .011	3	− .022:	K0.0	+6.0	+5.0	+5.9
568	439	....	5.78	4.56	+ .032	5	.....	G8.5	+1.3	+1.4	+1.3
582	450	6.45	5.84	5.08	− .100	4	− .109	G1	.....	.....	+4.1
635	495	6.19	5.74	4.85	− .111	3	− .117	F9.0	+2.7	.....	+3.1
694	542	....	5.49	4.06	+ .024	4	.....	G5	.....	+0.1	+0.5
720	561	....	5.90	5.08	− .116	4	.....	F9	+4.4	.....	.....
754	589	5.89	5.02	3.86	− .015	3	− .012	AG5	.....	+1.3	+0.5
800	619	....	5.58	4.40	.000	4	− .032:	cG0.5	−3.0	.....	−2.1
885	672	....	5.61	4.37	+ .036	4	.....	G4.5	+0.8	−0.3	+0.2
931	704	6.94	5.84	4.49	+ .037	3	+ .048	G8.0	+1.7	+0.7	+0.9
937	710	4.70	4.17	3.31	− .120	4	− .115:	G1.0	+4.3	.....	+3.9
995	751	....	5.94	4.89	− .005	5	.....	G5	.....	+1.1	+2.5
996	752	5.62	4.95	4.00	− .078	5	− .076:	G3.5	.....	+4.0	+4.7
1007	763	6.76	5.76	4.63	− .006	3	.000	G8	.....	+2.7	+0.6
1101	825	....	4.40	3.56	− .122	5	.....	F9.7	+4.0	.....	+3.9
1115	835	6.58	5.76	4.49	+ .009	3	+ .005	G5.5	+1.6	+1.0	+0.8
1176	876	....	5.79	4.69	− .029	3	.....	G0	.....	.....	+1.5
1283	952	6.88	5.67	4.35	+ .044	3	+ .058	K1.0	+1.0	+1.1	+0.4
1327	988	....	5.40	4.35	− .021	3	.....	G3	.....	+0.9	+0.9
1343	999	....	5.10	3.93	+ .007	3	.....	G9	.....	+2.6	+0.6
1360	1008	6.86	5.90	4.74	− .002	2	+ .001	G6	.....	+1.5	+0.7
1407	1042	6.42	5.29	3.71	+ .076	3	+ .088	K1.0	+1.6	+0.2	+0.5
1535	1134	....	5.79	4.54	+ .027	4	.....	K0	.....	+2.3	+0.9
1603	1185	5.04	4.22	3.02	− .005	3	− .018:	cG0.3	−3.0	.....	−2.5
1729	1259	5.39	4.85	3.95	− .114	5	− .105:	G1.0	+4.6	.....	+4.4
1780	1294	5.61	5.14	4.34	−0.136	6	− .135	F8	.....	.....	+3.7

TABLE 8—*Continued*

Star		Magnitudes			Color	N	Mean		Absolute Magnitudes		
BS	PGC	Ptg.	Vis.	Infr.	S_c		Color	Sp.	M_v	M_c	M_w
1963	1407	$6^m.21$	$5^m.24$	$3^m.61$	$+0^m.096$	4	$+0^m.089$	K1	.....	$+0^M.2$	$+0^M.2$
2014	1441	6.80	5.89	4.81	− .027	4	− .021	G3	.....	+0.9	+0.9
2251	1582	6.40	5.81	4.97	− .085	2	− .096	G0.0	$+3^M.4$	.....	+3.7
2331	1627	....	5.96	4.78	+ .013	4	.....	G8.0	+2.1	+2.1	+2.5
2333	1628	6.72	5.77	4.45	+ .041	3	+ .035	G8.5	+0.9	+1.2	+0.3
2427	1694	....	5.09	3.47	+ .107	4	.....	K1.5	+0.4	0.0	+0.4
2477	1720	....	5.47	4.28	+ .013	4	.....	G6.0	+1.7	+1.0	+0.9
2483	1724	....	5.34	4.55	− .132	4	.....	F9.0	+4.4	.....	+4.0
2569	1780	....	5.77	5.01	− .120	3	.....	G0	.....	.....	+4.4
2660	1822	....	5.60	4.48	+ .012	4	.....	G3.5	+1.0	+0.1	+1.4
2721	1860	....	5.55	4.78	− .108	4	.....	F8.9	+5.0	.....	+4.1
2720	1865	6.79	5.56	4.10	+ .087	3	+ .093	K0	.....	−0.1	+0.3
2808	1921	....	5.07	4.01	− .019	3	.....	G5	.....	+1.6	+1.0
2828	1936	6.19	5.07	3.93	+ .012	3	+ .020	G5	.....	+0.2	+0.3
2924	1997	....	5.61	4.56	− .007	2	.....	G5	.....	+1.1	+0.8
2939	2006	....	5.77	4.47	+ .030	4	.....	G6	.....	+0.4	+0.9
3176	2146	....	5.38	4.54	− .096	3	.....	G3	.....	+4.6	+4.1
3182	2150	....	5.48	4.28	+ .019	3	.....	G4	.....	+0.2	−0.6
3191	2157	....	5.83	4.79	− .023	3	.....	G5.5	+3.2	+2.1	+4.2
3216	2174	....	5.73	4.57	+ .005	3	− .009:	G6	.....	+1.8	+0.9
3264	2205	....	5.93	4.60	+ .056	2	.....	K1.5	+1.1	+1.5	+0.5
3303	2231	....	6.01	4.84	+ .013	3	.....	G7	.....	+1.2	+0.8
3323	2247	4.24	3.47	2.40	− .029	3	− .033:	AG2.0	.....	+0.6	+0.5
3391	2284	....	5.69	4.86	− .105	3	.....	G0.5	+4.9	.....	+4.2
3400	2289	....	5.74	4.55	+ .034	4	.....	G6	.....	+0.3	+1.4
3508	2368	....	5.24	4.04	+ .023	4	.....	G6	.....	+0.6	+0.7
3540	2391	....	5.25	4.17	+ .016	3	.....	G6	.....	+0.9	+0.4
3575	2409	....	5.64	4.22	+ .000	3	.....	G9	.....	+3.0	+0.3
3612	2437	5.83	4.71	3.45	+ .021	2	+ .026:	G4.0	−1.2	0.0	−0.5
3621	2444	....	5.38	4.44	− .019	2	.....	G7	.....	+2.7	+0.6
3664	2478	....	6.02	4.97	− .014	3	.....	G3	.....	+0.7	+2.3
3743	2530	....	5.56	4.30	+ .023	4	.....	G5.0	+0.4	+0.1	+1.0
3771	2549	5.49	4.57	3.64	− .019	2	− .029	G1.7	+3.4	.....	+3.7
3782	2555	6.15	5.12	3.87	+ .026	3	+ .028	K1	.....	+2.0	+0.6
3800	2566	5.65	4.62	3.54	+ .001	3	− .001:	G5.7	+1.7	+1.3	+1.0
3873	2618	4.00	3.12	2.06	− .035	3	− .032:	cG1.5	.....	.....	−1.3
3881	2626	....	5.20	4.31	− .117	4	− .118:	G0.0	+4.2	.....	+3.8
3922	2660	....	5.99	4.93	− .002	2	.....	G5.0	+0.8	+1.0	+0.5
3951	2681	....	5.60	4.58	− .084	3	.....	G5.5	+4.9	+5.5	+4.8
4166	2829	5.55	4.77	3.75	− .021	3	− .035:	cG1.0	−0.7	.....	−1.6
4235	2890	....	5.76	4.47	+ .104	3	.....	K1	.....	−0.3	+0.4
4277	2920	....	5.14	4.25	− .105	3	− .098:	G0.5	+4.4	.....	+4.2
4319	2950	6.60	5.66	4.38	+ .033	3	+ .027	AG7	.....	.....	+2.6
4392	2995	....	5.06	3.85	+ .040	2	+ .053:	G7	.....	−0.1	+0.1
4400	3000	6.47	5.52	4.37	+0.019	2	+0.010	G6.0	+0.5	+1.1	+0.8

TABLE 8—*Continued*

Star		Magnitudes			Color	N	Mean		Absolute Magnitudes		
BS	PGC	Ptg.	Vis.	Infr.	S_c		Color	Sp.	M_v	M_c	M_w
4407	3007	....	5^m85	4^m64	$+0^m036$	3	.....	G7.5	$+0^M3$	$+0^M7$	$+0^M9$
4457	3046	....	5.76	4.52	+ .054	2	.....	G8	.....	+0.4	+0.7
4495	3074	....	5.43	4.13	+ .021	5	.....	K1	.....	+2.2	+0.6
4496	3075	....	5.46	4.44	− .038	5	-0^m049:	G6.0	+5.7	+4.0	+5.2
4608	3155	5^m20	4.24	3.10	+ .030	4	+ .025:	G6.0	−0.4	+0.5	+1.1
4643	3181	....	5.67	4.51	+ .025	5	.....	G6.5	+1.1	+0.7	+1.0
4785	3279	4.82	4.32	3.47	− .115	3	− .118:	G0.7	+4.6	.....	+4.2
4883	3347	....	5.07	4.08	− .067	6	.....	F5	.....	.....	+2.3
4929	3381	6.98	6.01	4.88	+ .028	5	+ .015	G7	.....	+1.1	+0.5
4983	3424	4.84	4.32	3.52	− .101	6	− .115:	G1.3	+4.0	.....	+4.5
5011	3443	5.82	5.22	4.46	− .120	4	− .120	F9.5	+4.0	.....	+3.7
5072	3487	5.71	5.16	4.13	− .062	6	− .063:	G4.5	+3.8	+3.9	+4.4
5091	3497	....	5.94	4.86	− .042	2	.....	G4.5	+2.0	+2.7	+0.5
5149	3533	....	5.80	4.56	+ .041	4	.....	G9.0	+0.8	+1.1	+0.8
5183	3554	7.02	6.32	5.58	− .114	3	− .110	G0.5	+4.4	.....	+3.8
5330	3655	6.41	5.36	4.18	+ .043	4	+ .032	G6	.....	+0.3	+1.0
5420	3715	....	5.61	4.55	.000	3	.....	G3.5	+0.8	+0.5	+2.7
5480	3753	5.95	5.03	3.75	+ .022	3	+ 020	G5.0	+0.9	+0.2	−0.3
5481	3754	6.59	5.63	4.48	+ .042	4	+ .022	G6.5	+0.4	+0.8	+0.6
5544	3798	5.37	....	3.60	− .021	3	.....	G6, K5	.....	.....	.....
5635	3856	....	5.21	4.23	+ .011	2	.....	G4.5	+0.8	+0.5	+0.9
5691	3893	....	5.23	4.49	− .133	3	− .138:	F9	.....	.....	+3.1
5692	3894	....	5.66	4.61	+ .017	2	.....	G5	.....	+0.3	+0.5
5694	3895	5.67	5.18	4.36	− .152	3	− .139	F6.5	+4.0	.....	+3.3
5785	3958	....	5.88	4.68	+ .023	2	.....	G5	.....	+0.1	+1.1
5823	3979	....	5.41	4.20	+ .006	3	− .016:	G5.5	+2.4	+1.8	+0.8
5840	3992	7.12	5.97	4.98	− .017	2	− .004	G5.0	+1.3	+1.0	+0.2
5853	4000	6.71	5.80	5.04	− .067	2	− .067	G5	.....	+4.1	+4.4
5868	4010	5.12	4.42	3.68	− .121	3	− .107:	G0.3	+4.6	.....	+4.1
5889	4024	5.43	4.73	3.64	− .041	2	− .046:	G5.0	+1.8	+2.9	+1.6
5911	4040	6.82	6.16	5.30	− .105	2	− .099	G0.5	+4.8	.....	+3.7
5914	4042	5.16	4.61	3.82	− .105	5	− .114	F8.3	+4.5	.....	+3.6
5966	4075	6.35	5.28	4.02	+ .026	3	+ .032	G6.0	+1.0	+0.3	+0.7
6132	4192	3.78	2.89	1.67	+ .003	3	+ .001:	AG6.3	+1.8	+1.5	+0.7
6152	4207	....	5.29	3.76	+ .167	4	.....	G4.5	−1.7	−2.8	+0.6
6183	4228	....	5.95	4.72	+ .030	4	.....	G6	.....	+0.4	+0.3
6199	4240	....	5.44	4.11	+ .057	4	.....	G9.0	+1.2	+0.5	+0.9
6212	4246	3.47	3.00	1.97	− .081	4	− .092:	AG0.0	+3.2	.....	+3.7
6458	4403	....	5.36	4.56	− .081	4	.....	G0.5	+4.2	.....	+4.7
6536	4443	3.74	2.99	1.69	+ .020	3	+ .005:	AcG0.3	−2.0	.....	−1.7
6608	4488	....	5.72	4.96	− .063	5	.....	G5.0	+2.7	+3.8	+1.2
6623	4497	4.11	3.48	2.53	− .058	3	− .065:	G4.0	+3.6	+4.0	+4.2
6770	4580	5.76	4.73	3.61	+ .011	4	+0.010	G8.0	.....	+2.2	+0.4
6791	4589	....	5.11	3.95	− .001	5	.....	G5.5	+2.0	+1.2	+0.7
6807	4601	....	5.87	4.54	−0.001	2	.....	G7.0	+1.1	+1.8	+0.5

TABLE 8—*Continued*

Star		Magnitudes			Color	N	Mean		Absolute Magnitudes		
BS	PGC	Ptg.	Vis.	Infr.	S_c		Color	Sp.	M_v	M_c	M_w
6866	4635	5^m88	4^m92	3^m79	$+0^m002$	3	$+0^m001$	G5.5	.....	$+1^M1$	$+0^M4$
7117	4782	....	5.38	4.19	+ .009	4	.....	G6.5	$+1^M4$	+1.2	+0.6
7123	4887	....	5.62	4.54	− .027	6	.....	G8.0	+4.0	+4.3	+4.7
7137	4799	5.90	4.97	3.89	+ .003	4	− .005	G3.0	.....	+0.4	+0.8
7162	4815	....	5.21	4.45	− .079	4	.....	AF9.5	+5.0	.....	+4.6
7218	4848	....	5.52	4.54	− .030	4	.....	G3.0	+0.3	+1.2	+0.9
7293	4892	....	5.97	5.04	− .078	4	.....	AG4.5	+4.7	+4.7	+4.7
7300	4898	6.86	5.69	4.38	+ .035	5	+ .049	G7	.....	+0.0	+0.5
7373	4950	6.06	5.23	4.32	− .077	4	− .067	G6.5	+4.6	+5.2	+4.5
7382	4958	....	5.95	4.81	+ .002	3	.....	G5.0	+0.8	+0.8	+0.6
7468	5012	....	5.16	4.14	+ .019	3	.....	K0	.....	+2.8	+2.4
7479	5023	5.16	4.37	3.47	− .050	4	− .055:	cF8.0	+0.2	.....	−1.4
7506	5039	....	5.45	4.46	+ .006	3	.....	G6	.....	+1.2	+0.5
7560	5065	5.76	5.22	4.39	− .127	4	− .122	F9.0	+3.6	.....	+3.9
7606	5096	....	5.89	4.89	− .059	3	.....	F6	.....	.....	+2.1
7672	5146	6.46	5.89	5.03	− .101	3	− .105	G2.0	+4.4	4.0	+4.4
7741	5192	....	5.38	4.03	+ .027	5	+ .025:	cG5.5	−1.8	(0.3)	−2.3
7743	5193	....	5.78	4.63	+ .020	3	.....	G6	.....	+0.7	+1.5
7747	5197	....	4.55	3.08	+ .065	3	.....	cG3.5	.....	−1.2	−2.7
7796	5229	3.10	2.32	1.42	− .099	4	− .082:	cF7.0	−3.2	.....	−2.1
7896	5304	5.99	5.23	4.19	− .072	6	− .065:	G2	.....	+1.8	+3.8
7955	5344	5.18	4.63	3.81	− .126	5	− .125:	F8.0	+3.6	.....	+3.0
7995	5373	5.54	4.76	3.55	− .012	5	− .014	G3.7	+0.9	+1.1	+0.9
8232	5527	....	3.07	1.96	− .030	4	− .020:	cF9.0	−2.6	.....	−2.5
8313	5590	5.66	4.52	3.11	+ .069	5	+ .072	cG4.3	−0.4	−1.2	−1.3
8548	5786	6.40	5.85	4.97	− .123	4	− .115	F6.5	+3.8	.....	+3.5
8618	5840	6.87	5.80	4.79	+ .020	3	+ .010	G6.0	+0.9	+1.1	+0.6
8631	5851	6.60	5.81	4.80	− .046	5	− .046	AG3	.....	+1.9	+4.2
8729	5917	6.17	5.59	4.63	− .078	3	− .082	G2.5	+4.1	+3.3	+4.3
8752	5931	....	5.48	3.62	+ .174	3	+ .111:	cG3.0	−4.0	−2.2	−3.2
8807	5959	6.37	5.56	4.31	+ .010	5	+ .003	G4	.....	+0.8	+2.4
8905	6024	5.14	4.57	3.63	− .095	4	− .101:	F6.3	+0.5	.....	+2.8
8916	6037	5.48	4.45	3.09	+ .057	3	+ .053	G9.3	+0.4	+0.8	+0.5
9088	6172	....	5.85	4.71	− .023	4	.....	AG3.0	+5.2	.....	+4.7
BD	61178	....	6.40	3.89	+ .356	1	.....	cK5	.....	−4.6	−3.6
Polaris		....	2.10	1.19	− .156	1	.....	cF7.0	−2.2	.....	−2.2
BD	53722	....	7.40	5.59	+ .163	1	.....	cG2	.....	−3.6	−2.0
C	2163	7.17	6.02	4.84	+ .022	1	+ .030	K1.5	+3.1	+2.3	+5.6
C	2279	6.61	5.94	5.26	− .103	3	− .112	F9.0	+4.5	.....	+3.9
C	2495	6.93	5.99	5.16	− .049	3	− .052	G4.5	+3.9	+(3.3)	+4.9
PGC											
	391	....	3.65	2.54	(− .043)	5	.....	G5.5	+6.0	+3.2	+5.6
	848	....	3.72	2.45	+ .019	6	.....	K0.0	+4.6	+2.8	+4.6
	2573	....	5.48	4.47	− .062	4	− .046:	G9.5	+5.9	+6.0	+5.3
	5009	....	4.78	3.76	− .044	4	− .042:	G8.5	+5.7	+5.3	+5.6
	5093	4.61	3.90	2.74	−0.016	3	−0.025:	G7.3	+3.4	+3.4	+4.0

SOME CHARACTERISTICS OF ASSOCIATED GALAXIES

I. A DENSITY RESTRICTION IN THE METAGALAXY

E. F. CARPENTER

ABSTRACT

The densities of distribution of galaxies in 42 metagalactic groups and clusters have been computed. The fundamental data were drawn chiefly from descriptions published at the Harvard and Mount Wilson observatories. Distances of the latter set have been correlated with the red-shift, while distances of the former are entirely photometric, but results from both sources are consistent. Analyses of partly observed memberships of clusters are believed to furnish estimates of the total membership with sufficient accuracy for this investigation.

The study shows that the smaller a cluster is, in dimensions and membership, the greater its density may be. The distribution of clusters in the density-volume diagram is bounded by the fairly well-defined envelope $\rho_{max} = 4400/\sqrt{V}$, and suggests that the conventionally recognized clusters of galaxies are not separate metagalactic entities but are merely the extremes of a nonuniform space distribution conditioned by the above-mentioned density restriction. The foregoing value of the constant of proportionality results from the adoption of -14.2 as the average absolute magnitude of a galaxy.

One of the distinctive features of the distribution of the galaxies in space is the tendency to associate in groups which range all the way from pairs to clusters of many hundreds of members. This tendency appears superficially to resemble the clustering characteristics of the stars (with the exception of the globular clusters), and a study of it seems to offer an opportunity to elucidate some of the reactions of galaxies upon their neighbors.

For convenience, the discussion of binary galaxies is deferred to a later paper. The present discussion concerns only the larger associations of galaxies, the groups and clusters. The terminology in use in this connection is, however, not uniform. By some, the term "cluster" is applied to all groups of galaxies, large or small, which appear real; by others it is reserved for the great collections of several hundreds of galaxies. It will be indicated in this paper that there is no essential distinction between the large, rich clusters and the smaller groups, the total populations and other characteristics varying gradually from one extreme to the other.

For the most part, investigations of the clusters have been concerned with the determination of the luminosity function of the

members and the consequent establishment of the distance and the velocity-distance relationship. On account of the labor of gathering the data, comparatively little information has accumulated regarding the physical structure of the clusters. A. Wallenquist[1] has analyzed the distribution of density in the Coma cluster, and Sinclair Smith[2] has more recently demonstrated the absence of rotation of the Virgo cluster and inferred a state of equilibrium for the cluster as a whole. Somewhat earlier Harlow Shapley[3] computed the densities of 25 groups, most of them in the southern hemisphere, and concluded that "we can take 5×10^{-30} galaxies per cubic centimeter[4] as the approximate mean density of matter in space for a large number of groups of galaxies, and note that in a few relatively small groups, where the individual galaxies are nearly in contact, the mean density may be ten times as great." G. Lemaître[5] has reduced Shapley's material anew and used it with more complete data from Mount Wilson on eight other clusters to support the "collapse" process of his cosmogony.

In 1931 the writer[6] suggested a simple functional relationship between the diameters and the populations of clusters, on the basis of the seven clusters for which adequate information was then available. The data were meager though consistent, but the formula was realized to be incompatible with the existence of small, compact groups of galaxies, now recognized abundantly. The appearance of the descriptions of 25 groups by Shapley and of the cluster in Hercules found at the Steward Observatory[7] provided a basis for the alteration of the functional relationship to an expression of a density restriction or mass restriction in space,[8] and M. L. Humason's[9] more recent and nearly complete descriptions of eight new

[1] *Ann. v. d. Bosscha-Sterrenwacht*, **4**, 73, 1933.

[2] *Mt. W. Contr.*, No. 532; *Ap. J.*, **83**, 499, 1936.

[3] *Proc. Nat. Acad.*, **19**, 591 and 1011, 1933; *Harvard Reprints*, Nos. 92 and 102.

[4] This equals 15,000 galaxies per cubic megaparsec. On the assumption of $\bar{M} = -14.2$, in conformity with this paper, this density becomes 22,000 galaxies per cubic megaparsec.

[5] *Proc. Nat. Acad.*, **20**, 12, 1934.

[6] *Pub. A.S.P.*, **43**, 252, 1931.

[7] Carpenter, *Pub. A.A.S.*, **8**, 44, 1934.

[8] *Pub. A.A.S.*, **8**, 108, 1935. (By title.)

[9] *Mt. W. Contr.*, No. 531; *Ap. J.*, **83**, 10, 1936.

clusters and groups are sufficiently confirmatory to warrant an attempt to put the matter into an approximate quantitative form.

THE DATA

The data are shown in Tables 1 and 2. Forty-two clusters and smaller groups are represented. The Mount Wilson data are taken from the papers of Hubble and Humason and are reduced on the

TABLE 1

THE MOUNT WILSON DATA

CLUSTER	NUMBER OF MEMBERS			DISTANCE (MPS.)	DIAMETER (MPS.)	DENSITY (THOUSANDS OF NEB. PER CU. MPS.)
	Observed	F*	Estimated			
Coma	800	1.0	800	17	0.49	13
Virgo	500	1.0	500	2.2	.41	14
Perseus	500	1.0	500	12	.40	15
Corona Borealis	400	1.0	400	44	.39	13
Leo	300	1.0	300	38	.40	8.8
Ursa Major I	300	1.0	300	31	.38	10
Ursa Major II	200	1.5	300	86	.26	22
Gemini I	200	1.0	200	42	.37	7.8
Boötes	150	1.5	225	84	.37	5.8
Cancer I	150	1.0	150	11	.21	32
Perseus	100	1.0	100	8.7	.15	55
Pisces	25	1.0	25	8.4	.15	15
Group near NGC 68	24†	1.0	24	14.‡	.15	14
Group near NGC 83	20	1.0	20	13.‡	.080	73
Group near NGC 7320	5*	1.0	5	14.‡	0.016	2,300

* F is the ratio of the estimated to the observed memberships.

† Lundmark (*Ark. f. Mat., Astron., o. Fys.*, **20A**, No. 13, 1927) reports a cluster of 150 galaxies here, but even if all these are physical members of this cluster and are confined to the area 36′ in diameter given by Humason for the 24 dominant members, the resulting sixfold increases in membership and density do not carry this cluster beyond the envelopes of Figs. 1 and 2.

‡ The distances, etc., of these three clusters are based entirely upon the red-shift.

basis of -14.2 as the adopted value of the mean absolute magnitude, $\overline{M}$, of an individual galaxy. The distances of the small groups are derived from the velocity-distance relation, which is established chiefly by the larger clusters. For all but the two most remote of the Mount Wilson clusters the counts of the populations are taken as essentially complete. For those two, since the most frequent apparent magnitude corresponding to their distance is about 21.0 (corrected for the effect of the red-shift), the counts are

assumed to be about two-thirds complete (col. 4). In the computation of the mean densities the clusters were taken as spheres of diameters corresponding to their average projected diameters, irregularities of outline being necessarily ignored.

TABLE 2

THE HARVARD AND OTHER DATA

CLUSTER	NUMBER OF MEMBERS			MEAN MAGNITUDE	DISTANCE (MPS.)	DIAMETER (MPS.)	DENSITY (THOUSANDS OF NEB. PER CU. MPS.)
	Observed	F	Estimated				
Shapley No. 1	12	1.0	12	15.4	8.4	0.058	115
2	88	2.0	176	17.5	22	.38	6.0
3*	77	1.12	87	17.2	19	.22	16
4*	46	1.35	62	17.2	19	.22	10
5	56	1.25	70	16.4	13	.12	85
6	8	1.0	8	15.9	10	.031	540
7	56	2.0	112	17.2	19	.17	47
8	379	1.25	475	17.0	17	.51	7.0
9	46	1.1	51	16.5	14	.10	85
10*	58	1.5	87	17.3	20	.29	6.5
11*	45	2.5	115	17.3	20	.21	25
12*	39	1.12	46	17.7	24	.41	1.4
13	150	4.5	675	18.0	28	.56	7.4
14	370	4.5	1700	18.2	30	.77	7.0
15	256	5.5	1400	18.0	28	.59	13
16	99	2.5	250	18.0	28	.42	6.7
17*	157	1.15	180	17.4	21	.37	7.0
18	32	1.0	32	17.4	21	.12	34
19	21	1.0	21	16.3	13	.051	300
20	472	3.8	1800	18.5	35	.91	4.6
21	66	1.7	112	17.5	22	.23	18
22	58	5.0	290	18.5	35	.30	20
23	82	3.0	246	18.5	35	.60	21
24	26	2.5	65	18.5	35	.18	20
25	56	2.5	140	18.5	35	.32	8.1
Cnc II (Lundmark)	69	2.0	140	18.4	33	.12	170
Her I (Carpenter)	144	1.0	144	16.3	13	0.37	5.5

* For these six clusters two possible extreme estimates of F were made, with consequent changes in the dependent quantities of the table. The differences, referred to the larger value, averaged: in membership, 50 per cent; in diameter, 12 per cent; in density, 12 per cent. The means from the two values for each of these clusters are tabulated above.

The Harvard data are less complete, since the fainter members of most of the clusters lie beyond the magnitude limits of the plates. The complete populations were estimated by a procedure suggested by Lemaître,[10] who assumed the luminosity function of each group

[10] *Loc. cit.*

to be approximately an isosceles triangle on a base of five magnitudes, which is the total effective range of magnitude in the larger clusters which have been adequately surveyed. It seems hardly to be expected, however, that the groups in which only a few dozen members have been counted in a range of two or three magnitudes above the plate limits (e.g., Groups 9 and 12) would extend over the whole magnitude range of the richest clusters. Certain small groups (e.g., Nos. 1, 6, 19) whose faintest detectable members are well above the magnitude limits of the plates confirm this view. To take account of features of this sort the apparent luminosity function and hence the total membership and most frequent magnitude of each of the 25 Harvard groups were estimated from plots of the frequencies of apparent magnitudes as tabulated by Shapley. With the use of $\overline{M} = -14.2$ and Shapley's effective angular diameters (θ) the remaining quantities of Table 1 were computed as for the Mount Wilson clusters. Except for a few of the least completely surveyed groups, the results are of the same general order as those given by the purely formal method of Lemaître.

Since the few nebulae which constitute Group 24 exhibit a rather indecisive distribution of magnitudes, this group was reduced on the assumption of its being a physical companion of Group 25, as suggested by Shapley. Each of the other groups was reduced independently. Both members of the apparently double cluster made up of Groups 14 and 15 appear to be equally distant, but in the apparently triple cluster composed of Groups 17, 20, and 21, only 17 and 21 agree as to distance.

Two other clusters are available. Cancer II was described by Lundmark,[11] who found it on a Mount Wilson negative. Its characteristics, as listed in Table 2, have been derived from Lundmark's magnitude-frequency distribution by the method used for the Harvard clusters. Unfortunately, the zero-point of the magnitude scale could be established only by star counts, so that the physical description of this cluster must be attended by relative uncertainty. Hercules I was found and explored at the Steward Observatory, the magnitude scale being established by comparison with Selected Area 36, but the description given in Table 2 is based on an unpublished revision of the counts of the field nebulae.

[11] *Lund Obs. Circ.*, **1**, 30, 1931.

THE INTERPRETATION

The data of the tables may be conveniently displayed by a plot of the logarithmic mean densities, log ρ, against the linear diameters, Δ, of the clusters (Fig. 1). The Mount Wilson data are rep-

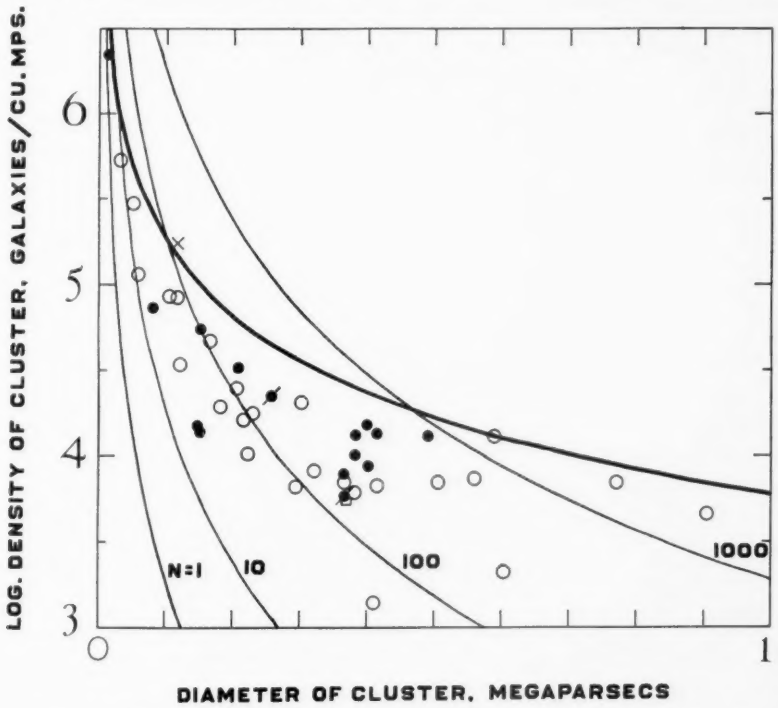


FIG. 1.—The densities and diameters of 42 clusters of galaxies. The abscissae indicate the diameters, Δ, of the clusters, expressed in megaparsecs. The ordinates indicate the logarithm of the mean density, ρ, of each cluster, expressed as the number of galaxies per cubic megaparsec. The sources of the data are distinguished as follows: Harvard, open circles; Mount Wilson complete counts, disks; Mount Wilson incomplete counts, barred disks; Lundmark, cross; Carpenter, rectangle. The heavy curve is the envelope $\rho_{max} = 6040\,\Delta^{-3/2}$. The four lighter curves represent clusters having memberships $N = 1$, 10, 100, and 1000 galaxies.

resented by disks, the two clusters for which the total populations have been estimated being distinguished by bars. The Harvard data are represented by open circles, Cancer II by a cross, and Hercules I by a rectangle. The light lines indicate various total populations, N.

The plotted points are strikingly characterized by a tendency

to keep to the lower and left parts of the diagram, except that the corner is avoided. This avoidance is undoubtedly a matter of selection, for points in this region represent very small groups of low density not readily distinguishable from and probably identical with the ordinary fluctuations of nonuniform space distribution. For example, a point at $\Delta = 0.1$, $\log \rho = 3.76$ represents three average nebulae in a circle $2^{\circ}.1$ in diameter if they are of the thirteenth magnitude, or in a circle $0^{\circ}.8$ in diameter if they are of the fifteenth magnitude. Cases like these are to be found by the hundreds in the nebular catalogues, and a considerable proportion of them must by physical rather than merely apparent groups. On the other hand, points in the upper right-hand part of the diagram would represent very large, dense, and rich clusters, which certainly ought to have been found if they exist.

We may therefore bound the plotted points by an envelope (full line, computed on the basis of Fig. 2) and interpret it as defining a density restriction in metagalactic space such that a given density of galaxies may extend over no more than a certain limited volume which is larger for smaller densities, or, conversely, such that a cluster of given extent may have no more than a certain limited density which is larger for the smaller clusters.

Since the envelope cuts across the lines of equal population, it defines a population or mass restriction. This may be evaluated from the plot of Figure 2, which shows the logarithmic populations and logarithmic diameters of the same clusters, indicated as in Figure 1. The envelope is evidently a straight line, which may be drawn close to the extreme lower left disk, for this represents the well-known Stefan quintet, whose members are intricately involved, so that a smaller group diameter is not likely for its membership. Similarly, the three circles at the extreme right represent unusually rich clusters. The equation of the envelope so drawn is $N_{max} = 3160\ \Delta^{3/2}$, where N_{max} is the maximum possible membership of a cluster Δ megaparsecs in diameter. Then, if ρ_{max} is the maximum possible density, expressed in galaxies per cubic parsec,

$$\rho_{max} = 6040\ \Delta^{-3/2}\ .$$

We may also write, where V is the volume of a cluster in cubic megaparsecs,[12]

$$N_{\max} = 4400\ V^{1/2}\ ; \qquad \rho_{\max} = 4400\ V^{-1/2}\ .$$

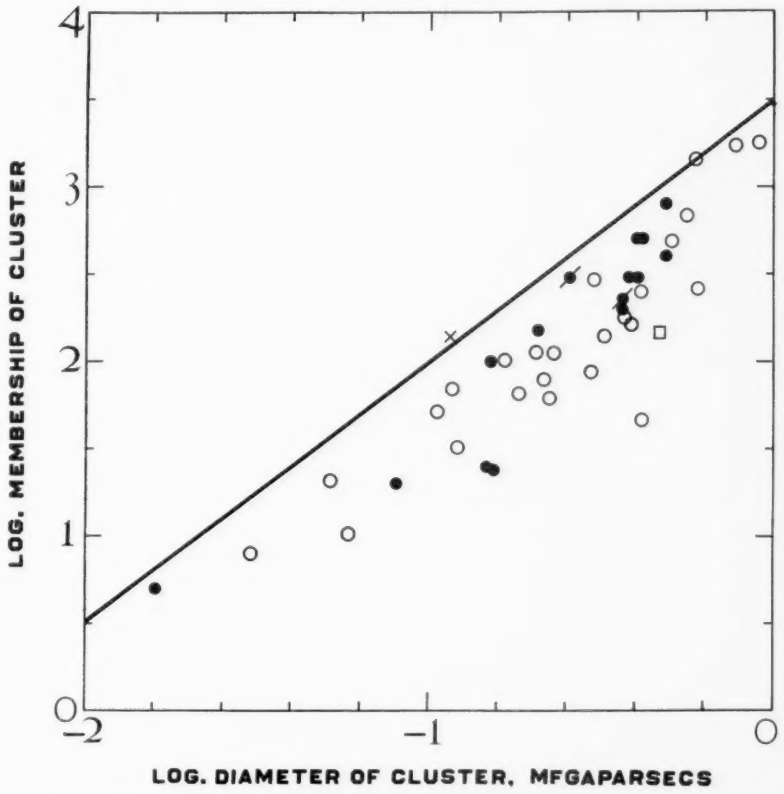


FIG. 2.—The memberships and diameters of 42 clusters of galaxies. The abscissae represent logarithms of the diameters, Δ, of the clusters, expressed in megaparsecs. The ordinates represent the logarithms of the memberships, N, of the clusters. The symbols for the data are the same as in Fig. 1. The straight line is the envelope $N = 3160\ \Delta^{3/2}$.

It is of some interest to compare the density restriction with Wallenquist's analysis[1] of the distribution of density in the Coma cluster, computed from the apparent surface densities based on

[12] If it is desired to use a value for the average absolute magnitude of a cluster galaxy which is ΔM different from the value used here (-14.2), each of these four numerical coefficients is to be multiplied by $2^{\Delta M}$.

Wolf's description[13] of the cluster. Estimating that 300 of the described nebulae are members of the cluster, Wallenquist tabulates the space-density at the center of the cluster and in the nine suc-

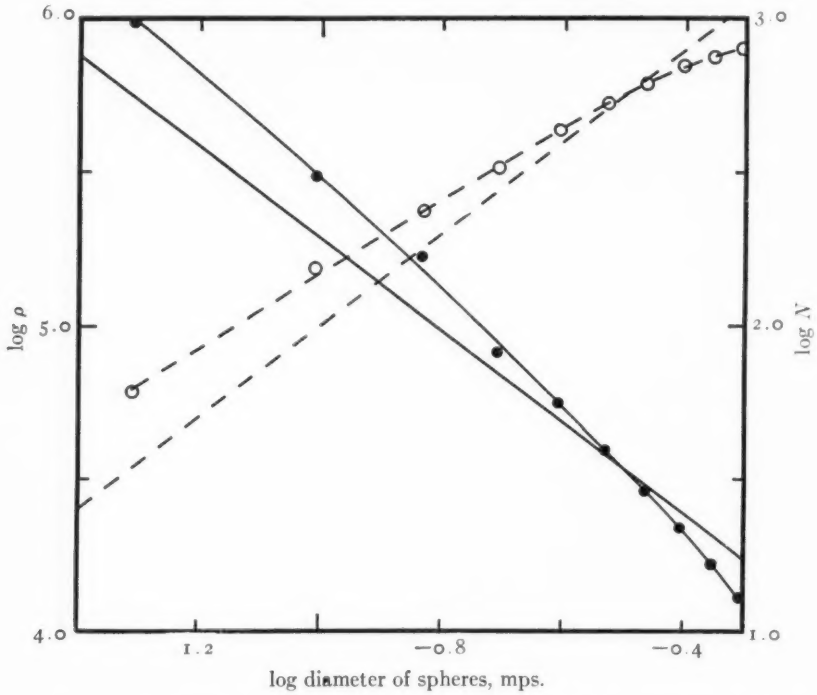


FIG. 3.—The internal structure of the Coma cluster of galaxies, compared with the density restriction and the membership restriction. The data are derived from Wallenquist's counts of 300 galaxies in ten spheres of equal increment of diameter within the cluster. The logarithms of the diameters of these spheres, expressed in megaparsecs, are plotted as abscissae. After adjustment for a total membership of 800 for the entire cluster, the logarithm of the count, N, within each sphere and the logarithm of the consequent mean density, ρ, are plotted as ordinates. The curved lines connect the plotted points; the straight lines represent the restrictions of density and membership for whole clusters of corresponding diameters. The solid lines refer to densities, the broken lines to membership.

cessive spherical shells 0.1 of a cluster radius in thickness. From this were computed the total population and mean density in each of the ten successive spheres of the cluster. On the assumption that

[13] *Veröff. Heidelberg*, **1**, 125, 1902. Cf. F. Zwicky, *Ap. J.* **86**, 217, 1937, which appeared after this paper had been essentially completed.

the 500 members of the cluster which are too faint for Wolf's list have the same distribution as the brighter members—a condition which is consistent with the behavior of the range of brightness available to Wallenquist—we may reduce these populations and densities to the total population of the cluster, 800, and compare them with the mass and density restrictions (Fig. 3). The restrictions appear to be violated in a significant manner, but this should not be unexpected, for the restrictions as derived apply to entire clusters whose densities essentially vanish at their effective surfaces, while in a sphere in the interior of a cluster the nebulae are more efficiently packed, the density at the surface of a small central sphere being still quite high.

SOURCES OF SYSTEMATIC ERROR

Certain possibilities of systematic error remain to be considered:

1. Partly on the basis of the two estimates of the most frequent magnitudes of the Virgo cluster, Lemaître suggests a systematic difference of about one magnitude between the Harvard and the Mount Wilson photometry. However, evidence from the direct comparison of magnitudes and from the consistency of nebular counts to various magnitudes, cited by Hubble,[14] indicates that any systematic difference that may exist is very much smaller than this. The apparently large difference in the Virgo results may be partly attributable to a difference in the elimination of the field nebulae of the Virgo cluster. In any case, the matter is not a serious one, so far as the establishment of the density restriction is concerned. A systematic difference of 0.5 mag. between the Mount Wilson and the Harvard photometry would, for example, result in the horizontal displacement of the disks or circles of Figure 2 by only 0.1 unit. In Figure 1 the displacement would be 26 per cent of the value of the abscissa to the right (or left) and 0.3 unit downward (or upward). Hence, neither the form nor even the amount of the restriction would be seriously altered.

2. The range of the density restriction is too large to be attributable to the reciprocal effects of erroneously established distances upon densities and diameters. In this connection, the Mount Wilson

[14] *Mt. W. Contr.*, No. 485; *Ap. J.*, **79**, 139, 1934.

data are not to be questioned, since the distances are co-ordinated with the red-shifts. Moreover, the evidence for the restriction from the Harvard data by no means depends only upon the completion of the magnitude frequency distribution curves as carried out here. It shows quite well in a qualitative way in Shapley's original minimum densities, and it appears even when, in order to eliminate this reciprocal effect, the distances are established purely on the hypothesis of constant linear diameter for the five largest members of each group,[15] although in this case some of the smaller clusters lose their very high density. This hypothesis is, however, rather unlikely, since it requires that the smaller groups be composed selectively of giant members, and this in turn suggests that the single, isolated nebulae should be larger (and brighter) than the cluster nebulae—a situation at variance with photometric and spectrographic data.

3. The estimated total populations of some of the Harvard clusters (especially Groups 14, 15, and 20) are rather large, particularly in view of Hubble's statement[16] that "no clusters with *as many as* a thousand members are definitely known." However, each of these three clusters contains from 250 to 470 observed members (the field nebulae being eliminated) in a range of only two magnitudes above the plate limits, so that either the total populations are of the estimated order or the luminosity functions depart widely from the pattern which is followed closely by the rich clusters which have been quite completely surveyed. But the point need not be unduly labored, for a factor of 2 either way in the estimated population changes $\log N$ or $\log \rho$ by only 0.3. Furthermore, the extremes of density have been assumed to have completely counted memberships. Hence any adjustments of total population would probably enhance rather than diminish the selective nature of the density restriction.

4. Finally, to test whether the population restriction might be due to some obscure effect of selection, the membership counts and linear diameters were examined for nearly 250 galactic star clusters,[17] the stellar analogues of the nebular clusters. Absence of any correlation between these two parameters was evident from the plot.

[15] *Proc. Nat. Acad.*, **19**, 1011, 1933. [16] *The Realm of the Nebulae*, p. 76, 1936.

[17] Shapley, *Star Clusters*, Appen. B, 1930.

While the fainter members of many of these clusters have not been counted, it seems unlikely that the counts would be so inadequate as to mask quite completely a structural feature of the kind and dominance sought, if it were present.

CONCLUSIONS

It is concluded that the density restriction and the mass restriction in metagalactic space are real and represent a fundamental property of the distribution of matter in space. It follows as a corollary and from the interpretation of Figure 1 that there is no basic and essential distinction between the large, rich clusters and the small, loose groups. Rather, the objects commonly recognized as physical clusterings are merely the extremes of a nonuniform though not random distribution which is limited by density as well as by population. From this point of view, the term "supergalaxy" is of questionable propriety, since it implies a distinctive and coherent organic structure inherently of a higher order than the individual galaxies themselves.

It is difficult to regard these restrictions as consequences of ordinary principles of dynamics. Consider, for example, n identical clusters, each one characterized by $N = N_{\max} = kV^{1/2}$. If these should intermingle, the assembly would have to expand to the volume n^2V in order to satisfy the observed mass restriction. If $N = 100$ and $n = 10$ (which give configurations within the range of data), the rather extreme nature of the readjustment is evident.

One might, of course, expect to find a *minimum* membership (or density) limit for each value of Δ, consequent upon the requirements of dynamic stability of the cluster. But the data suggest no evidence for such lower limits. On the contrary, if a cluster near the envelope (Fig. 2) is just dense enough to be stable, then all the other clusters of the same membership situated to the right of it on the diagram must be unstable unless the mean kinetic energy per unit mass is systematically *smaller* in the *larger* clusters. Difficulties of this sort appear to suggest that the observed maximum restrictions are intimately related to the mode of origin of the clustering of galaxies.

STEWARD OBSERVATORY
UNIVERSITY OF ARIZONA
March 1938

NOTES

THE SPECTROSCOPIC TRIPLE STAR 59 d SERPENTIS

ABSTRACT

This star is composed of a close pair of nearly equal stars of class A having a period of about 2 days, and a distant G-type companion having a probable period of 15 years. The spectroscopic triple is the brighter component of a wide visual double.

The spectrum of 59 d Serpentis is classified in the *Henry Draper Catalogue* as a composite of Ao and G. The star is the brighter component of a visual double, No. 11353 of Aitken's catalogue, and the description of the spectrum leaves some doubt whether the composite character is referred to the integration of the two visual components or to the brighter component alone. Recent Michigan spectrograms, taken under excellent conditions, leave no doubt on this point; the composite spectrum clearly belongs to the brighter star, as the fainter was entirely off the slit during the whole exposure.

The first plate showed a spectrum dominantly of solar type in the region of longer wave length than *Hδ*, though the lines were somewhat weakened by the superposition of an early-type continuous spectrum. The hydrogen lines and *Mg* II 4481 appear abnormally strong for a solar-type spectrum, and they are doubtless much affected by the early-type stars. At the K line of *Ca* II there is a narrow and strong line. The second spectrogram was taken 6 days later and shows the same characteristics, except for a distinct doubling of the K line and a suspicion of duplicity at λ 4481. Plates on succeeding days showed K double with slightly unequal components which exchange places on alternate days, so that the period is close to 2 days. Finally, 13 and 14 days after the first observation the K line was again single. The period is therefore close to either 1.87 or 2.15 days.

The solar-type spectrum is either class F8 or class Go, and the early-type binary is probably composed of two stars of class Ao

or A2. Effects due to the lines of the binary are certainly present at *Mg* II 4481 and *Sr* II 4078, though measurements of the binary components of these lines are difficult and inaccurate.

The radial velocities obtained to date are given in Table 1. Except as noted, the values for the A-type stars depend on the K line alone. They are individually subject to uncertainties of several kilometers, but a probable center-of-mass velocity of the close binary is indicated at about −8 km/sec.

TABLE 1

RADIAL VELOCITIES OF 59 d SERPENTIS

DATE (G.C.T.) 1938	VELOCITIES IN KM/SEC		
	G star	A stars	
		Bright	Faint
July 6.17	−29	−20	.....
12.15	37	+65	− 85
13.15	49	−92	+ 83
14.19	39	+74	−110
15.21	47	−79	+ 97
17.15	52	(−73)*	.....
19.14	35	− 8	.....
20.13	−30	− 6	.....

* Velocity based on Mg II 4481. The plate was underexposed.

The G-type star has shown no large fluctuations and is evidently a distant companion. The mean value of its velocity, −40 km/sec, is comparable with the minimum observed at Yerkes Observatory[1] in 1905 and 1906. In all probability the binary and the G-type star now exhibit nearly their maximum difference of velocity. The Yerkes and Lick[2] observations indicate a probable period in the neighborhood of 15 years. Shorter periods cannot be reconciled with the available data.

The velocities of the G-type star show a scatter which is too great to assign to errors of observation when we consider the quality of the lines. It is noticeable that the highest (algebraic) velocities are associated with single lines of the binary and that the lowest occur

[1] *Ap. J.*, **30**, 66, 1909. [2] *Lick Obs. Pub.*, **16**, 269, 1928.

when the stronger component of the binary has a large negative velocity. There can be little doubt that we are dealing with an effect of blending of lines of the binary with those of the solar-type star. At the time of measurement occasional notes were made of faint shadings and asymmetries of lines which fit this interpretation.

A rough value of $m \sin^3 i$ of the triple system may be calculated from the 15-year period and the relative velocity of the G-type star and the binary. The figure 15–20 times the sun seems unexpectedly large and may argue for a shorter period. The value of $(m_1 + m_2) \sin^3 i$ for the binary is only about 1.3. This points to an inclination very far from 90°. It is thus impossible to account by eclipses for the suspected variability of the star. The G-type star, however, is obviously a giant, and it may be an irregular variable.

Attempts to obtain observations which will clearly distinguish between the two possible periods have so far been defeated by the weather. Observations of this interesting system are being continued. The writer would greatly appreciate any information concerning radial velocities of the G-type star during the long interval 1915–38.

DEAN B. MCLAUGHLIN

THE OBSERVATORY, UNIVERSITY OF MICHIGAN
July 23, 1938

A NOTE ON THE SPECTRUM AND RADIAL VELOCITY OF γ PERSEI

ABSTRACT

The K line of *Ca* II in this composite spectrum shows variations in velocity opposite to those of the solar-type spectrum. A period of about ten years is suggested.

The spectrum of γ Persei is an excellent example of a composite spectrum. According to the *Henry Draper Catalogue*, the classes of the components are F5 and A3. Spectrograms obtained at the University of Michigan Observatory show that the class of the brighter star is probably G0; the classification as F5 doubtless resulted from the influence of the hydrogen lines, which are intensified by the early-type component. Aside from this effect, the photographic spectrum of longer wave length than $H\epsilon$ appears to be of a pure solar type.

The K line, however, is strong and very sharp, as in class A2 or A3, and it obviously belongs to the secondary star.

Radial velocities obtained previously have referred only to the solar-type component. Extreme values obtained during the years 1897–1911 at Columbus,[1] at Bonn,[2] and at the Lick Observatory[3] are +8.6 and −3.0 km/sec. Five Michigan spectrograms give the velocities shown in Table 1. It will be seen that the K line undergoes variations of velocity opposite to those of the Go spectrum. This variation of relative velocity was first detected, previous to measurement, when the first two plates were examined together on the Hartmann spectrocomparator.

TABLE 1

RADIAL VELOCITIES OF γ PERSEI

DATE, U.T.			VELOCITIES IN KM/SEC	
			Go Star	A3 Star (K Line)
1932	Oct.	16.26	+21.1	−39
1937		17.23	− 0.6	0
	Dec.	1.23	− 2.2	+ 7
1938	July	30.39	− 1.2	+ 6
	Aug.	9.40	− 0.9	+ 6

The earlier data from other observatories indicate a broad minimum of velocity for six or seven years at about −2 km/sec, and the recent Michigan plates evidently refer to that part of the curve. A slow decline from 1904 to 1906 suggests that a sharp maximum occurred in 1903, when no observations were obtained. The observation in 1932 was evidently obtained, by fortunate chance, at another such sharp maximum. Thus, the period can have one of only three values: roughly, ten, fifteen, and twenty-nine years. The shortest period appears the most likely. The period of fifteen years is not ruled out, but the longest period would require improbably large masses. The observations will be continued.

DEAN B. MCLAUGHLIN

THE OBSERVATORY, UNIVERSITY OF MICHIGAN
August 12, 1938

[1] *Ap. J.*, **21**, 314, 1905. [2] *Ibid.*, **27**, 305, 1908. [3] *Lick Obs. Pub.*, **16**, 38, 1928.

DIMENSIONS OF THE SOLAR GRANULES

ABSTRACT

From observations in visual light the average diameter of the larger granules which are ordinarily observed is estimated at 1".4, or 1100 km. Their centers are spaced about 1500 km apart, so that they occupy about 35 per cent of the surface area of the sun. Their total number at any moment is roughly 2,600,000.

The precise formulation of any theory to account for the granular structure of the visible surface of the sun requires a knowledge of the scale of the granulation. From visual observations Huggins,[1] Langley,[2] and others estimated that the granules were usually from 1" to 2" in diameter, with smaller ones probably also present. These conclusions were confirmed by the early photographic work of Janssen,[3] Hansky,[4] and Chevalier.[5] However, recent photometric investigations by H. H. Plaskett[6] in several spectral regions and by ten Bruggencate, Grotrian, and von der Pahlen[7] in the violet suggested that the surface structure is on a larger scale, with particles of the order of 5" across. On the other hand, Strebel[8] has confirmed the earlier observations while working with visual light, but finds a coarser structure on his ultraviolet images, which may correspond to low-level spectroheliograms.

Thus there remains some inconsistency between the published observations. For this reason measurements have been made on several negatives taken recently through the 40-inch objective, diaphragmed to small apertures, at moments when the air was steady enough to give unusually fine definition. The exposures were made on plates coated with Eastman Kodalith emulsion, type Va-G, especially prepared for us through the kindness of Dr. C. J. Staud, of the Eastman Kodak Company. It is due primarily to their high contrast and extremely fine grain (the diameter of the developed plate grains

[1] *M.N.*, **26**, 260, 1866.

[2] *Ibid.*, **34**, 255, 1874; *Amer. J. Sci. and Arts*, **9**, 192, 1875.

[3] *Ann. de l'Obs. d'astr. phys. de Paris* (Meudon), **1**, 103, 1896.

[4] *Pulkovo Mitt.*, **1**, 81, 1905; **3**, 1, 1908.

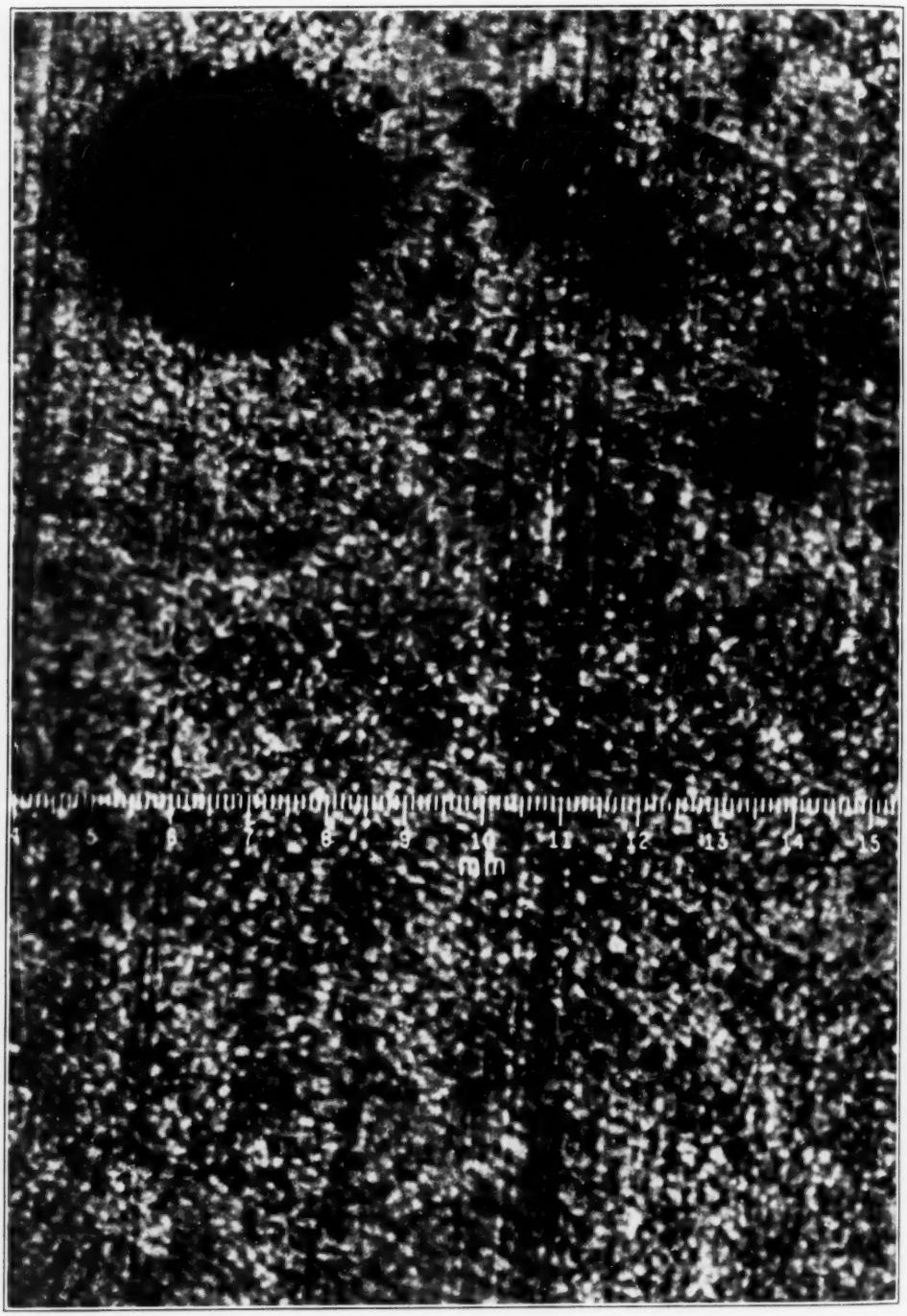
[5] *Ap. J.*, **27**, 12, 1908. *Zô-Sé Ann.*, **8**, Part C, 1912.

[6] *M.N.*, **96**, 402, 1935.

[7] *Zs. f. Ap.*, **12**, 323, 1936; **16**, 51, 1938. Dr. ten Bruggencate has since informed me by letter that his recent observations in the blue region of the spectrum show the 1"–2" granulation clearly.

[8] *Ibid.*, **6**, 313, 1933.

PLATE XV



Solar Granulation on July 10, 1937

UNIV. OF MICH.

was estimated at 0.3 μ under a magnifying power of 2300) that these plates were better suited for measurement than those taken here in past years under equally good observing conditions.

The exposures were made through various combinations of green and yellow filters, which, together with the sensitivity-curve of the panchromatic emulsion, confined between 5200 and 5800 A the spectral region employed. Throughout this range the focal curve of the telescope is nearly flat, and the effective wave length may be taken as 5550 A.

One of the negatives exposed on July 10, 1937, proved to be the best yet obtained, and was selected for detailed measurement. A portion of this image near the center of the sun's disk is reproduced in Plate XV. The millimeter scale was enlarged in contact with the negative and can be used to estimate directly the sizes of the granules. At the visual focus of the 40-inch, 1 mm corresponds to 10".66. The vertical streaks in the picture are due to specks on the filters, which were carried on the moving slit of the focal-plane shutter.

On the reproduction it is easy to distinguish the areas of good definition from the streaks and spots where the solar details are blurred by convection currents in our atmosphere. In order to determine the frequency and average size of the individual granules, two of these areas of best definition were chosen, and within them all distinguishable granules were counted and measured with a measuring-engine in two co-ordinates on the original negative. In Table 1 the second column gives the factor by which the projected area must be multiplied to reduce it to the center of the disk, the third column the corrected area, the fourth column the total number of granules, and the fifth column the arithmetical mean of their diameters. In some cases it was not possible to decide whether the object measured was one irregular large granule or a clump of two or more closely adjacent small ones. A more significant mean diameter is obtained if these doubtful cases are omitted, and this has been done in forming the seventh and eighth columns.

To make certain that this plate was representative of normal conditions, 72 well-defined granules were measured on a plate taken on July 19, 1938. The average diameter found was 1".64; but the close coincidence with the other values is not very significant, for

the regions of good definition on this plate were so small that it was necessary to pick out scattered granules for measurement, and there may be some effect of selection. In any case the areas used on the 1937 plate appear to be fair samples of the sun's surface.

TABLE 1

DIMENSIONS OF GRANULES

Region	Cos θ	Area	All Granules		Clearly Defined Granules	
			Number	Mean Diameter	Number	Mean Diameter
1	0.971	3.68×10^8 km^2	153	1".67	136	1".61
2	0.974	5.51	241	1.69	200	1.60

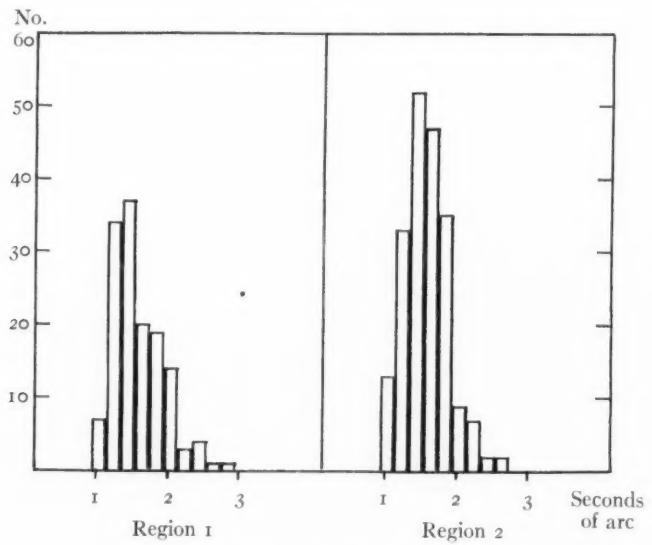


FIG. 1.—Frequency distribution of diameters

Frequency plots of the data in Table 1 are shown in Figure 1. The absence of diameters of less than about 1" is almost certainly due to two observational factors: atmospheric scintillation and finite resolving-power of the telescope. The atmospheric blurring is greatly reduced by using small apertures, but the improvement that can

be gained in this way is limited by the aperture for which the diameter of the central diffraction disk becomes comparable with the size of the images of the granules. For the plate of July 10, 1937, which was taken through a 19-cm diaphragm, the distance from the center of the image of a point source to the first dark ring is given by the usual diffraction formula as $0''.7$. If the effective radius of the central image is taken as half this amount, we should expect that the diameters of the granules would be overestimated by about $0''.7$. However, the amount of structural detail which the plate shows in even the $1''.5$ granules suggests that the actual correcting factor which should be applied is considerably less, perhaps $0''.2$ or $0''.3$.

TABLE 2

ABUNDANCE OF GRANULES

Region	Number per 10^8 Km^2	Total Number on Sun	Average Distance between Centers
1.....	41.6	2,520,000	1,550 km
2.....	43.8	2,650,000	1,510
Mean.....	42.7	2,580,000	1,530 km

Although the lower part of the frequency-curves is thus distorted by limited resolution, so that nothing can be said about the presence or absence of very small granules, the diagrams establish the important fact that the larger granules ordinarily observed have diameters ranging from $1''$ to $2''$, with a maximum frequency near the mean value of about $1''.4$. A comparatively small number range in size from $2''$ to $3''$, which represents an upper limit which is rarely exceeded by an individual granule.

The number of granules per unit area can be found from the counts, and is given in the second column of Table 2. These numbers, multiplied by the sun's surface area, 6.06×10^{12} km^2, give rough values for the total number of granules on the surface of the sun at any moment, as shown in the third column. The fourth column contains the mean distance between the centers of individual granules, computed on the assumption of uniform distribution with the relation $1'' = 735$ km.

Since in linear measure the diameters found range from 710 km to 2100 km, with the mean at about 1100 km, the fraction of the sun's total surface area which is occupied by the bright granules is approximately 35 per cent. However, the remaining space is not a uniformly dark background, for much of it is occupied by matter of intermediate brightness, probably the remains of granules which had formed a few minutes before but which, at the moment of observation, were already so far dissolved as to be unrecognizable. This irregularly distributed debris can be seen in the reproduction but shows to much better advantage on the original negative, on which the contrast is not so great.

These data fully confirm the conclusions of the early visual and photographic workers as to the scale of the granulation observed in green and yellow light. It is highly desirable that further observations should be made with large reflectors to determine accurately how the apparent size of the granules varies with wave length.

YERKES OBSERVATORY
September 1938

PHILIP C. KEENAN

EMISSION NEBULOSITIES IN CYGNUS AND CEPHEUS*

ABSTRACT

Several extended regions in Cygnus and Cepheus are enveloped in diffuse nebulosities which show *Hα* and [*O* II] 3727 in emission. These nebulosities are usually not concentrated toward individual bright stars, and it is probable that they derive their light from the ultraviolet radiation of the integrated O and B stars in Milky Way star clouds. The faintest nebulosities observed require about 10 atoms of *H* in the third quantum level, in a column of 1 cm² cross-section. A rough computation shows that this interstellar hydrogen has a density corresponding to 0.01 atom per cubic centimeter.

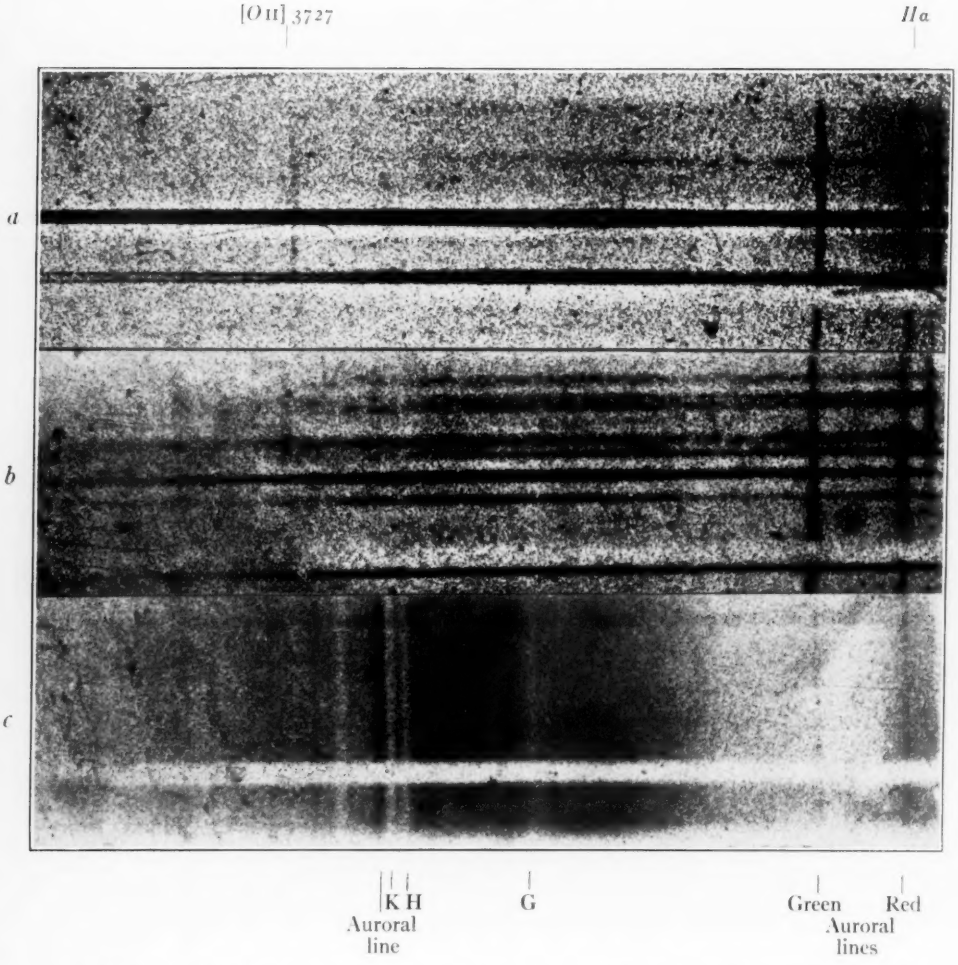
Several spectrograms have been obtained with the nebular spectrograph of the McDonald Observatory,[1] for the purpose of finding new emission nebulosities.

1. Guiding star +39°4082. 1938 Aug. 18/19. Exp. 2^h41^m. Comparison mirror $\Delta\delta = +20°$. *Hα* and [*O* II] 3727 conspicuous in emission over the entire length of the slit. This is doubtless an exten-

* *Contributions from the McDonald Observatory, University of Texas*, No. 9.

[1] *Ap. J.*, **87**, 559, 1938.

PLATE XVI



a) Emission nebulosity in Cygnus; *b*) emission nebulosity in Cepheus; *c*) spectrum of sky in twilight showing that region of *Hα* and of [*O* II] 3727 are normally equally strong in the main spectrum and in the comparison spectrum.

UNIV. OF MICH.

sion of the emission nebulosities near γ Cygni observed by Greenstein and Henyey.[2] The definition is not good enough to show Barnard's dark nebula, B 343, which crosses the field.

2. Guiding star $+39°4054$. 1938 Aug. 28/29. Exp. 4^h20^m. Comparison mirror $\Delta\delta = 0°$. *Hα* and [*O* II] 3727 conspicuous in emission. This region lies southwest of the preceding one, in the Cygnus star cloud. It may be a remote extension of the γ Cygni nebulosities.

3. Guiding star 28 b^2 Cygni. 1938 Aug. 20/21. Exp. 5^h. Comparison mirror $\Delta\delta = +20°$. *Hα* and [*O* II] 3727 conspicuous in emission. The intensity is greatest just west of b^2 Cygni, and it diminishes toward the east but apparently extends over the entire length of the slit (45′ east and 22′ west of the guiding star). This region is too far removed from γ Cygni, and the emission must be stimulated by the group of O and B stars in the Cygnus cloud.[3]

4. Guiding star 25 Cygni. 1938 Aug. 21/22. Exp. 4^h. Comparison mirror $\Delta\delta = +20°$. *Hα* and [*O* II] 3727 fairly conspicuous in emission. This region shows no definite nebulosity on the photographs by Barnard and by Ross. It is located in the Cygnus star cloud.

5. Guiding star $+29°3829$. 1938 Aug. 22/23. Exp. 3^h. Comparison mirror $\Delta\delta = +25°$. *Hα* looks slightly stronger in the Milky Way field than in the comparison field, but the evidence is not conclusive; [*O* II] 3727 seems to be absent. This region is located in the southern part of the Cygnus star cloud.

6. Guiding star π^1 Cygni. 1938 Aug. 23/24. Exp. 5^h30^m. Comparison mirror $\Delta\delta = +20°$. There may be a slight emission at *Hα*, but the evidence is not conclusive.

7. Guiding star $+55°2714$. 1938 Aug. 24/25. Exp. 5^h42^m. Comparison mirror $\Delta\delta = +16°$. This region looks slightly nebulous on Ross's photograph (*Atlas*, No. 19). *Hα* and [*O* II] 3727 are very conspicuous in emission; H_β and the blend of N_1 and N_2 are visible. The star $+55°2721$, classified by Beals[4] as O7 + OW7, is in the field, being about 2′.5 south of the guiding star. The emission lines are strongest near this star, but *Hα* is bright from slightly west of the

[2] *Ibid.*, **86**, 620, 1937.

[3] Struve, *A.N.*, **231**, 17, 1927; Bok, *The Distribution of the Stars in Space*, p. 117, 1937.

[4] *Pub. Dom. Ap. Obs., Victoria*, **4**, 286, 1929.

guiding star to the eastern limit of the plate. The emission fades out behind the obscuring nebulosity west of the guiding star.

8. Guiding star $+54^{\circ}2708$. 1938 Aug. 25/26. Exp. 4^{h}. Comparison mirror $\Delta\delta = +16^{\circ}$. *Hα* and [*O* II] 3727 fairly conspicuous in emission. This nebulosity is related to the preceding one, although there are no conspicuous O stars or early B stars in the field.

9. Guiding star δ Cephei. 1938 Aug. 26/27. Exp. 6^{h}. Comparison mirror $\Delta\delta = +17^{\circ}$. *Hα* seems to be definitely present in emission, but the line is weak and quite uniform over the full extent of the slit. [*O* II] 3727 is probably present in emission.

10. Guiding star λ Cephei. 1938 Aug. 27/28. Exp. 6^{h}. Comparison mirror $\Delta\delta = +15^{\circ}$. *Hα* is in emission, extending uniformly, though faintly, over the full length of the slit. [*O* II] 3727 is concentrated toward the star, although it seems to extend as a faint line to the limits of the slit.

The diffuse nebulosities observed in Cygnus and Cepheus cover very large areas and, with one exception, show little or no tendency to concentrate toward a bright star. In this respect they differ from Hubble's nebulae. It is probable that they are excited by the general stellar radiation coming from the O and B stars in Milky Way star clouds. The emission ceases rather abruptly in high galactic latitudes: our comparison fields were about 10° to 20° north of the galactic equator. A very rough comparison suggests that the faintest emission of *Hα* recorded by us in a six-hour exposure has about the same density as a two-second exposure on the central parts of the Orion nebula. From a polar sequence plate by Dr. Seyfert, Dr. Keenan found for Schwarzschild's exponent, $p = 0.760$. Hence, the ratio of the intensities is about 10^{3}. For the Orion nebula Ambarzumian[5] has determined the numbers n_4 and n_5, of fourth-level and of fifth-level hydrogen atoms in a radial column having a cross-section of 1 cm^2. From his data we conclude that the number of third-level hydrogen atoms in the same column is approximately

$$n_3 = 10^{4} \text{ (Orion)} .$$

[5] *Zs. f. Ap.*, **6**, 112, 1933.

In our faint nebulae we have, consequently,

$$n_3 = 10 \text{ (Cepheus and Cygnus)} .$$

Evidently, the excitation is not uniform; it depends upon the concentration of O and B stars. Consider a region in the Milky Way where this concentration is normal. We neglect self-absorption and assume that we can see to distances of 10^3 parsecs. Dark nebulae presumably cut off our vision beyond that distance. Hence, the number of third-level *H* atoms per cubic centimeter is

$$N_3 = 3 \times 10^{-21} .$$

It is reasonable to suppose[5] that the right order of magnitude for the relative population of *H* in the third quantum level is given by

$$\frac{N_3}{N_1} = \frac{g_3}{g_1} W e^{-h\nu/kT} .$$

The dilution factor *W* can be obtained, for interstellar space, from data by Greenstein.[6] We shall use $T = 25{,}000°$ and $W = 3 \times 10^{-17}$. Hence,

$$\frac{N_3}{N_1} = 3 \times 10^{-19} .$$

This would give us 10^{-2} for the total number of *H* atoms per cubic centimeter. It is quite likely that we are here dealing with ordinary interstellar hydrogen. Observations of interstellar absorption lines give 10^{-7} atoms of *Ca* II per cubic centimeter. If the fraction of singly ionized *Ca* atoms is 10^{-3}, we find that the total number of *Ca* atoms per cubic centimeters is 10^{-4}. Hence, the interstellar abundance of *H* by atoms is about 10^2 times that of *Ca*. These computations are quite uncertain, but they give an indication of the nature of the faint nebulosities.

If we are dealing with a star cloud which is rich in O and B stars,

[6] Unpublished. These data depend upon an integration over all stars of all spectral types. Black-body radiation was assumed. Interstellar absorption was taken into account. A large excess of ultraviolet radiation would alter the results.

we may take $W = 3 \times 10^{-16}$. In that case the extent of the nebula is also smaller, perhaps only 10^2 parsecs. The final abundance of H comes out the same.[7]

OTTO STRUVE
C. T. ELVEY

McDONALD OBSERVATORY
AND
YERKES OBSERVATORY
September 1938

[7] In a paper presented at the Ann Arbor meeting of the American Astronomical Society, Dr. V. M. Slipher criticized some of the results obtained with the Yerkes and McDonald nebular spectrographs. (1) He asserted that the results on the Pleiades by Greenstein and Henyey are not in accordance with his own earlier observations and that new spectrograms show his former results to be correct. There is no discrepancy between Slipher's observations and those of Greenstein and Henyey. We have always emphasized that the Yerkes nebular spectrograph integrated over a region of 16′ diameter, while our present spectrograph integrates over 6′ if focused on the slit, or over 0′ if focused on the sky. The fundamental nature of Slipher's early observation has always been recognized by us. The Yerkes observation was for the purpose of showing how the early-type absorption spectrum gradually merges with the general sky spectrum. Had there been conspicuous emission lines in the outer regions, they would have been found on these plates. The contamination by starlight referred to by Slipher is slight, but the blurring of the features of the objects is important and has been fully considered. (2) Dr. Slipher suggested that the nebula north of Antares is not red and is not illuminated by Antares. Our photovisual and photographic observations (*Ap. J.*, **84**, Pls. V and VI) prove that the nebula is indeed red, compared with nearly all other reflection nebulae. Hence, it is probably illuminated by Antares. There seems to be a tendency for reflection nebulae to be slightly bluer than their respective stars (*ibid.*, **86**, 529, 1937), and this may account for Slipher's observation. The only serious discrepancy of which we are aware, between our work and that of Slipher, is his conclusion with respect to the nebulosities in Orion that ". . . . the variation in the spectrum may be said to begin in the center with the usual emission type which loses strength (different substances differently) with distance outward and ends finally, in the most distant masses, with a continuous spectrum of the normal stellar absorption type" (*Pub. A.S.P.*, **31**, 215, 1919). The observational evidence which led to this conclusion was doubtless sound, and the conclusion itself perhaps justifiable when this work was done; but we believe that the detection at the Yerkes and the McDonald observatories of emission spectra in the outer regions of Orion, near γ Cygni, and in the star clouds of Cygnus and Cepheus has amply justified a similar search in the outer regions of the Pleiades and in the nebula north of Antares.

REVIEWS

The Fundamental Principles of Quantum Mechanics. By EDWIN C. KEMBLE. New York: McGraw-Hill Book Co., 1937. Pp. 611. $6.00.

This is the type of treatment of the principles of quantum mechanics for which many of us have been waiting. It is less elegant but much more detailed than either the book of Dirac or that of von Neumann; it is mathematically much more rigorous than Dirac's treatment, and is much more readable and easily understood than that of von Neumann. As the author states in his Preface, he has tried to bridge the gap between the exacting technique of von Neumann and the usual less rigorous formulations of the theory. This he does without assuming knowledge of any but elementary mathematics on the part of his readers.

It should be stated at the outset that this treatment is definitely not suitable for an introduction to quantum mechanics—it is probably not suitable as a textbook for any course on the subject, for who would have his students work through hundreds of pages of mathematical discussion of principles with practically no treatment of specific physical problems? Rather is it a complete and extremely valuable reference work on the methods of quantum mechanics, with the material set down in as readable a form as one could ask considering the carefulness of the mathematical treatment.

Particularly noteworthy is the completeness of treatments of boundary conditions, the Sturm-Liouville problem, the Wentzel-Kramers-Brillouin method, matrix and operator theory, the different sorts of perturbation theory, and the continuous spectrum. I believe this is the first detailed treatment of the continuous spectrum to appear in any book in English. In most of the book Schrödinger's equation is taken as fundamental, the matrix and transformation theories being looked upon as tools to aid in its solution.

The physical arguments are very easy to follow, and are expressed with a notably apt choice of terms. One might mention in particular the commendable discussion of the meaning of ψ, the wave-particle dualism, and the significance of the uncertainty of quantum theory in connection with causality and determinism. The discussion may be followed by reading selected paragraphs on pages 4, 5, 6, 7, 76, 77, 86, 243, 244, 327, 328, and 331–334. A few sentences might be quoted here:

The writer would prefer to regard both waves and corpuscles ultimately as mental aids in the description and prediction of empirical results, leaving all questions regarding their objective reality to the philosophers [p. 4].

There is no real reason for supposing that a particle belonging to an assemblage described by a wave function has at every moment a definite, but unknown, position which may be revealed by a measurement of the right kind, or a definite momentum which can be revealed by a different measurement. On the contrary, we get into a maze of contradictions as soon as we inject into quantum mechanics such concepts carried over from the language and philosophy of our scientific ancestors. As no scheme of operations can determine experimentally whether physical quantities such as position and momentum exist and have unique values when they are not at the moment under observation, nor whether the number obtained by a measurement describes some objective property of the thing measured, a strict adherence to the operational point of view requires that we eliminate such concepts from our theories. From the standpoint of classical physics such a rejection would perhaps be a bit of philosophical purism flying so unnecessarily in the face of common sense that few would care to adopt it. On the other hand, this rejection is a demonstrable logical necessity for quantum mechanics [p. 244].

The wave function of a pure-state assemblage is merely a tool for computing from all previous observations what the relative probabilities are for different results when we make our next observation. . . . . That the wave function is merely a subjective computational tool and not in any sense a description of objective reality [is shown by the fact that] if the observer forgets the result of his observation, or loses his notebook [the wave function is altered] [mélange of pp. 331 and 328 for which I apologize].

Only a few very simple physical problems are considered, and electron spin is not introduced until page 491, in a chapter on "Atomic Structure." While the statistical mechanics of quantized systems is discussed in some detail, the book does not carry the treatment to the point of introducing Fermi-Dirac and Bose-Einstein statistics. With this we have no complaint, since these subjects are carefully treated by Fowler and others. But this serves as an illustration of the subject matter which the book is intended to cover: it omits all detailed applications to special topics, since in many cases these are thoroughly covered elsewhere. But the writer of a treatise on a special topic does not have room for more than a brief sketch of the large body of fundamental theory of which he makes use, and this book supplies this material admirably.

Professor Kemble has been at work upon this book (I believe more or less constantly) since the appearance of his *Reviews of Modern Physics* articles in 1929, and the carefulness and thoroughness of its treatment are

ample evidence of and justification for the unstinted time and thought which have been devoted to it.

GEORGE H. SHORTLEY

Ohio State University

Glossary of Physics. Compiled and edited by LEROY D. WELD. New York: McGraw-Hill Book Co., 1937. Pp. vii+255. $2.50.

In 1933 the division of physical sciences of the National Research Council appointed a committee on a glossary of physical terms. The chairman of this committee, Professor LeRoy D. Weld of Coe College, has prepared the glossary, with the efficient aid of a large group of special consultants. A few astronomical terms have been included, but apparently no attempt has been made to give even a moderately representative list of them. We find a definition of the "light-year" but not of the "parsec" or the "astronomical unit." The terms "node" and "apse" are fully explained, but there is no mention of "periastron" or "perihelion." Nevertheless, the book should be very useful in a college or observatory library as a dictionary of the most common (but frequently bewildering) terms of modern physics.

S.

Lehrbuch der Stellarstatistik. By E. VON DER PAHLEN. Leipzig: Johann Ambrosius Barth, 1937. Pp. xv+934. Rm. 98.

Stellar statistics has only slowly acquired the status of a science. The reason is that as usually defined it has been, on the one hand, a research method and, on the other, a collection of numerical results derived from the primary data of observation. The research method is obviously a branch of applied mathematics; the numerical results, a branch of descriptive astronomy. As astronomy has progressed out of the mathematical stage which dealt with celestial mechanics and an enumeration of unrelated data concerning the fixed stars into a systematic description of the structural characteristics of star systems, stellar statistics has grown in importance and is now a very extensive part of the science of astronomy.

In spite of the vigorous growth of this new science in recent years, no general textbook has hitherto been written which covers the methods and results of stellar statistics. The contributions to its development are scattered in astronomical magazines and observatory publications. The methods used by research workers are often very simple, and the more involved methods have often been developed *ab initio* by the investigator.

The only technique the average student has learned is that acquired through his study of the method of least squares.

Professor von der Pahlen's textbook is the first comprehensive textbook in stellar statistics. The first part (pp. 142) deals with the fundamental principles and the methods of mathematical statistics in general. It was written by the late Dr. L. Hufnagel, who died in 1933 before his part was fully completed. This part describes the mathematical theory of distributions of "populations" of one or more dimensions and gives special emphasis to Charlier's method of moments and his A and B series. This part is of special value since it gives for the first time a general survey of modern mathematical statistics.

The second part (pp. 204) describes the astronomical data of observation. It is not in itself a part of statistical astronomy, although a thorough knowledge of the facts presented is necessary for the student. It tells how stellar positions and motions are derived, and how magnitudes, colors, and spectral types are defined and determined. It also describes trigonometric, photometric, and statistical parallaxes, the dynamic parallaxes being regarded as a special kind of trigonometric parallax.

The third part (pp. 334) deals with the space distribution of stars, star groups, and nebulae. The schematic distributions of Seeliger, Schwarzschild, and Kapteyn are here fully explained. This part will be of great value to many readers, because the original works are scattered among many publications and much of the pioneering work of Seeliger has been to a certain extent forgotten by modern investigators. Schwarzschild's elegant solution of the fundamental integral equations is described, and Kapteyn and van Rhijn's analysis is given in great detail.

The later, more empirical, studies of space distribution are fully described, in particular those by Pannekoek, van Rhijn, and Seares. The distributions of star clusters and of extragalactic nebulae as determined by Charlier, Shapley, Hubble, Lundmark, and many others are given in much detail.

The fourth and last part (pp. 233) deals with the kinematics and dynamics of stellar systems. It is the most complete account given anywhere of stellar motions and of the data on which the theoretical dynamics of the galaxy is based. An exposition of the theories of Oort and Lindblad on the rotation of the galaxy appears at the end of the book.

It is a monumental work which here is presented to the astronomical world. Its size and its scope probably make it unsuitable as a textbook on stellar statistics for use in colleges, but the exhaustiveness of treatment enhances its value for the research students. A very complete Index makes it particularly useful in this respect.

The author is to be congratulated for his successful completion of such a volume. He has himself written nine of the fourteen chapters, Dr. Hufnagel having contributed two and Dr. Gondolatsch of the Astronomische Recheninstitut three chapters.

GUSTAF STRÖMBERG

The Nature of Variable Stars. By PAUL W. MERRILL. New York: Macmillan Co., 1938. Pp. 134. $2.00.

Dr. Merrill of the Mount Wilson Obervatory, who is eminently qualified because of his own research to write on the subject of variable stars, has written a short, readable volume intended principally for the nontechnical reader. In addition to a general description of the varieties and physical characteristics of variables, there are a number of extracts from publications concerning early observations. Numerous firsthand research notes add to the interest of the volume.

W. W. M.

Photoelements and Their Application. By BRUNO LANGE. Translated by ANCEL ST. JOHN. New York: Reinhold Publishing Corp., 1938. Pp. 297. $5.50.

The title of this book requires an explanation since it introduces the word "photoelements" for the photoelectric cells discussed. This type of photoelectric cell is a spontaneous source of an electric current much as are the galvanic elements and the thermoelements, and hence the word "photoelement" is chosen to designate them. The general classification of photoelectric phenomena includes the external and the internal photoeffects. The external photoeffect is represented by the alkali photoelectric cells and the internal photoeffect by the photoelements or barrier-layer photocells and by the photoresistances or selenium type of cells.

The author divides the book into two parts. Part I is a historical and theoretical discussion of the photoelements as well as an exposition of the physical properties. This is a treatment which will be of interest to the scientist since he discusses the various theories to explain the phenomena. Part II is a survey of the present status of the technical uses of photoelements, written in quite simple style. In this section the author gives a working description of the barrier-layer photocells that will be of value to the experimenter who is interested only in the application of the photocell as a tool.

A glance through the book shows that a large part of the material presented consists of the author's own contributions, and justly so since he

was the rediscoverer of the phenomenon and one of the most productive workers in the field of photoelements.

The final chapter of the book contains brief references to the many applications of photoelements in the various scientific fields.

C. T. ELVEY

The Sun: Its Phenomena and Physical Features. By GIORGIO ABETTI. Translated by A. ZIMMERMANN and F. BORGHOUTS. New York: D. Van Nostrand Co., 1938. Pp. 360. $5.00.

This book was first published by the author in Italian (reviewed in this *Journal*, **83**, 508, 1936) and has now been revised and brought up to date for the English translation. The presentation is popular, and the book is intended not only for astronomers but also for workers in related fields. The observational results are described in considerable detail; and, as Dr. R. v. d. R. Woolley aptly remarks in hisPrefatory Note, " . . . . reference to [these] pages will remind astrophysicists of the existence of a large body of observational material which cries out for investigation by the inductive theorist." The illustrations, of which there are 157, are particularly well chosen.

S.

Physik der Sternatmosphären mit besonderer Berücksichtigung der Sonne. By A. UNSÖLD. Berlin: Julius Springer, 1938. Pp. viii+500. Figs. 145. Rm. 63; bound, Rm. 66.

The application of new physical theories to astronomical problems has always been the most fascinating branch of astrophysics. Hence, it is not surprising that Unsöld's new book, which is devoted entirely to the task of correlating physical theory and astrophysical observation, should be one of the most inspiring and stimulating treatises which the reviewer has read. Unsöld has avoided lengthy theoretical discussions and has freely mixed theory and application. "Astrophysical theories," he says in the Introduction, "which have lost their contact with reality are nothing more than unsuitable exercises in theoretical physics with which it would be better not to burden the pages of the scientific journals." He most emphatically does not propose "to write one more handbook of physics"; but instead he presents a thorough digest of what he, after careful deliberation, considers to be of permanent value in the vast storehouse of astrophysical publications.

The exposition is simple but in every respect complete. Any graduate

student can read the book from beginning to end; and while he may have to consult his textbooks in theoretical physics to refresh his memory on some points, the advance preparation required is really very slight—so slight, indeed, that it presents no problem. A student who has conscientiously worked his way through the entire book will not have a grasp of *all* branches of astrophysics; but in the particular field covered by the book—stellar atmospheres and the structure of the outer layers of the sun—he will possess a complete working knowledge of all the fundamentals required for independent research. The reviewer earnestly recommends that all students of stellar spectra read the book completely. Except for possible difficulties with the language, the serious student will find that the reading is easy, that the form of presentation, which introduces practical applications almost on every page, keeps his interest continuously alive, and that the frequent suggestions for new research open up before his eyes an almost unlimited field of exploration. With these advantages at his disposal he will readily forgive the author that he has occasionally been more individualistic—possibly even opinionated—than some other writer might have been. This feature is solely due to the fact that Unsöld himself has been one of the principal contributors to the scientific literature on the subject, and naturally he presents his own views where they disagree with others. The reader should have no difficulty, with the help of the very complete Bibliography at the end of the volume, to fill out any stages where he would prefer the opinions of other workers.

At any rate, whatever disadvantages may result from the strong personal participation of the author in the research which he presents, these are more than compensated by the many important new contributions which appear for the first time in this book. Among them are section 69, which gives the theory of the "curve of growth" for the important case of a stellar atmosphere with re-emission of radiation; section 71, which presents the practical application of the theory of collisional damping; section 72, which gives new data for the discussion of the Stark effect in the hydrogen lines; section 77, which discusses stellar rotation from a new point of view; and especially the concluding chapters, xvi and xvii, which are devoted to the sun.

The book consists of five parts. The first is devoted to the stellar atmosphere in thermodynamic equilibrium. The theory of radiation and the derivation and application to the sun of the Planck law, of the Stefan-Boltzmann law, and of the theory of thermal ionization are completely discussed. The second part is devoted to the continuous spectrum and

the structure of a stellar atmosphere. The third part lays the physical foundations for the theory of Fraunhofer lines. This is followed by Part IV on the measurement and interpretation of the contours of absorption lines. Finally, Part V deals with various problems and applications, such as the classification of stellar spectra, the interpretation of solar phenomena, the structure of the prominences, and the nature of the corona.

There is a short appendix on the classification of line spectra and another on the integral-exponential function. A Bibliography and two indexes, for names and for subjects, complete the volume.

Unsöld's discussion of solar phenomena is particularly valuable. Not only will astronomers engaged in solar research find in these chapters a complete theoretical discussion of the normal atmosphere of the sun, of sun spots, faculae, granules, and the corona, but they will find—probably for the first time in any book—a complete account of those observational data which are now required to answer the most pressing theoretical questions. A single example may illustrate this point. In section 98 Unsöld discusses, among other things, the self-absorption in the chromosphere. Since chromospheric emission lines are known to be affected by turbulent motions, he computes the self-absorption for an absorption coefficient which is determined by the Doppler effect of these turbulent velocities. A discussion of the observations by Pannekoek and Minnaert, and by Cillié and Menzel, shows that the true self-absorption in the chromosphere is much greater than the computed. Unsöld now suggests that the discrepancy may be explained if the chromosphere is regarded as a "burning prairie" of minute prominences. Within each prominence the velocities are Maxwellian, with an average $v = 1.2$ km/sec. The turbulent velocity of 15 km/sec represents the motions of the individual prominences. If in the line of sight we observe at any given instant through many little prominences, the Doppler constant in the absorption coefficient is 15 km/sec, as was actually used in the theory. But if we see through only a single prominence, the constant should be 1.2 km/sec. The great width of the chromospheric lines would then be the consequence of an integration, in space and in time, during the exposure. The difference would suffice to remove the discrepancy. Unsöld suggests that the question could be answered by means of visual observations of *He* D_3 in a solar telescope of large dispersion. If his interpretation is correct, the line should continuously vary in wave length (and in brightness) as the scintillation projects different prominence elements upon the slit.

It is not possible, in this review, to discuss in detail the extraordinary

variety of subjects covered in the various parts. In some cases Unsöld differs rather seriously from other workers. The large discrepancy between his and Russell's values for the hydrogen abundance in the sun is not fully explained, and the evidence presented by Unsöld is not entirely conclusive. Additional investigations will be required to settle this question. Unsöld's criticism of Milne's original theory of the *Ca* chromosphere is formally correct; but the substitute theory advanced by him, which assumes the existence of a large excess of radiation in the extreme ultraviolent region of the spectrum, over certain disturbed areas, leaves intact much of Milne's work, as well as of that of Chandrasekhar. It is not clear why Unsöld should regard Chandrasekhar's theory of the granulation as being in discord with the observations. After all, the principal premises of Chandrasekhar are substantially the same as those now advocated by Unsöld.

Unsöld was one of the pioneers in exploring the effects and causes of departures from thermodynamic equilibrium. Hence, it is difficult to understand why he should now be so skeptical about the matter as to pass over the theoretical considerations with a few remarks on pages 86 and 87. The entire first part of the book is under the spell of the thermodynamic equilibrium. Although his criticisms of too great a refinement in the formulae are entirely justified, it is dangerous to impress too rigidly the necessity of thermodynamic equilibrium conditions. The reader is quite likely to forget that these conditions are imposed not because we think that they hold in all stellar atmospheres but because we have not yet succeeded in perfecting the theory for other conditions. There is now ample evidence that in the outer atmospheres of many stars the dilution of radiation is tremendous. And while the existing theory is inadequate to cope with the problem completely, the theory of cycles by Rosseland certainly deserves attention. That Unsöld himself is not quite consistent in this matter is illustrated by his discussion of the departures from thermodynamic equilibrium in prominences, where he presents convincing evidence of a departure amounting to $10^{38.5}$ in the ratio He^{++}/He^{+}.

The book omits completely the field of band spectra. This is regrettable, partly because of the growing interest in this subject and partly because of the completely independent nature of the conclusions which may be made from band spectra concerning the physical conditions of a stellar atmosphere. Unsöld has also omitted the discussion of various emission-line stars, such as novae, *Be* stars, and late-type variables, of peculiar stellar spectra, and of diffuse nebulae. He refers to Rosseland's *Theoretical Astrophysics*, which covers some of these subjects.

In conclusion, no reader can fail to be impressed with the enormous amount of information which we owe in this field of knowledge to Karl Schwarzschild. Since some of Schwarzschild's original papers are difficult to obtain, a review of his classical investigations by Unsöld is especially welcome.

OTTO STRUVE

Materie im interstellaren Raume. By WILHELM BECKER. ("Fortschritte der Astronomie," Vol. I.) Leipzig: Johann Ambrosius Barth, 1938. Pp. iv+78. Rm. 7.50.

The secretary of the Astronomische Gesellschaft, Professor P. Ten Bruggencate, has undertaken the task of issuing, under the auspices of the Gesellschaft, a series of monographs under the title "Fortschritte der Astronomie." He intends to publish, on the average, one book per year. Dr. Becker's contribution is the first volume. Other issues in preparation are *Theories of Cosmology*, by Dr. Heckmann, and *Convection and Turbulence Effects in Stars*, by Dr. Siedentopf. The Astronomische Gesellschaft is to be congratulated upon having undertaken the publication of this series. As a continuation of the "Handbuch" it will be a valuable addition to astronomical literature.

Dr. Becker's monograph deals only with the problems of general and selective absorption in interstellar space. Interstellar line absorption is only briefly mentioned. The author has also omitted a detailed discussion of reflection nebulae and of emission nebulae—the former because they are at present being intensively investigated by several astronomers, the latter because they present a number of physical problems which cannot be adequately discussed in the limited amount of space available for the monograph. Both subjects are to be treated in future issues.

The most interesting chapter is the one on dark nebulae. The author discusses briefly the theory of Pannekoek, but does not carry through the derivation of the formulae necessary for a complete understanding of the practical procedure for finding the absorption, ϵ, and the distance, r, of the nebula. Although the use of Kapteyn's expressions for the luminosity function and for the density function can no longer be recommended, the method is very important for a thorough understanding of the entire problem. It forms the basis of Bok's numerical procedure and of Schalén's analytical methods. Moreover, for stars having a small dispersion of absolute magnitude, Pannekoek's method gives directly the required unknowns, ϵ and r.

It was, historically, a step in the wrong direction that long after the publication of Pannekoek's paper the practical astronomers were satisfied with a qualitative, and often incorrect, interpretation of Wolf diagrams. These diagrams constitute a useful basis for a theoretical discussion, but their value has only been realized through the application of analytical or numerical methods based upon Pannekoek's work.

Becker gives a number of striking comparisons. The primitive method gave for the region near ξ Cygni two clouds, at the average distances of 9 and 12 mag. stars. Miller's curves show only one cloud at the average distance of 13.8 mag. stars! Even more disturbing are the differences which result from the application of von der Pahlen's method, which uses for the normal regions the average values of $A(m)$ for the corresponding zone of galactic latitude. Thus von Klüber found for the Taurus region an absorption of 1 mag. by Wolf's method and of 2.5 mag. by von der Pahlen's method. It is often difficult to decide which method is better.

As Bok has pointed out in his astrophysical monograph, *The Distribution of the Stars in Space*, the dispersion in absolute magnitude is the main reason why a Wolf diagram with a complete break has never been observed. Hence, the accurate evaluation of the influence of the luminosity function is imperative in all discussions of dark nebulae. Becker fully supports this view, and his considerations should go far toward convincing the practical workers. If the two monographs by Bok and by Becker firmly establish this point, they will have served their purpose!

Becker discusses a number of errors which affect the results of the analysis. Of special interest is the effect of a scale error in the magnitudes. This point has usually been neglected, and it is important to have it discussed.

The final results are collected in a table and are shown in a diagram. The latter is rather confusing. The distances of the nebulae are plotted together with their galactic longitudes. However, distance zero is not in the center of the diagram, but on the periphery of a circle whose radius corresponds, on the scale adopted, to almost 200 parsecs. Hence, the visual impression from the diagram is badly distorted. The trouble is, of course, that if the distances are shown from the center of the diagram the markings would be crowded to such an extent that an interpretation would be difficult.

Becker stresses the possibility of a connection between the dark nebulae in Taurus and in Ophiuchus and Sagittarius. The evidence is, at present, rather slender. But the suggestion is interesting and stimulating.

The author expresses the hope that our knowledge of dark nebulae will be rapidly extended as a result of carefully planned investigations at Breslau, Harvard, Potsdam, Poulkovo, and Upsala. The reviewer, however, cannot escape the feeling that the routine methods now used for determining ϵ and r are still woefully inadequate. From an inspection of a long-exposure photograph an experienced observer can almost guess the values of ϵ and r. The complicated statistical discussions of star counts yield surprisingly little additional information, as is attested by the enormous differences resulting from the methods of von der Pahlen and of Wolf. We already know from the discussions of Bok and of Becker that the distances of the dark nebulae are often quite unreliable; that observers tend to infer the existence of two or more nebulae in the line of sight when an equally satisfactory representation of the observations may be obtained with one set of ϵ and r; and that there is a tendency to exaggerate the radial extent of the absorbing masses.

From the statistical theory we should expect that the values of ϵ are relatively reliable. Yet, a closer inspection raises a number of doubts. Becker mentions that von Klüber finds in the densest part of the Taurus nebula an absorption of 4 mag. An inspection of the photographs of Ross and of Barnard shows no background stars in the densest regions, and it is difficult to see how an accurate value of ϵ can at all be derived. The reviewer pointed out some time ago that the two small, luminous nebulae, B 10 and B 14, which are located in some of the densest parts of the Taurus cloud, depart from Hubble's relation by about 7 mag. Unless these nebulae are completely different in character from ordinary reflection nebulae, their exciting stars are dimmed by at least 7 mag. The dark clouds may well be considerably more opaque. A somewhat similar inconsistency occurs in Ophiuchus. Henyey has shown that the star $-24^{\circ}12684$, which is involved in the nebulae, is reduced in brightness by about 3.1 mag. The optical thickness of the dark cloud is almost certainly appreciably greater. Yet, the star counts give only 2.8 mag. for ϵ.

The discrepancy results from an inherent weakness of the method of star counts. The very dense regions are usually quite small and contain too few foreground stars for a statistical analysis. Most practical workers have tried to find ϵ and r simultaneously. Hence, they have chosen large areas and have averaged their star counts for regions of greater and lesser opacity. This procedure is beset with dangers. Let us suppose, for simplicity, that the obscured region consists really of two areas having ab-

sorptions of ϵ_1 and ϵ_2. We are dealing with a single nebula. Hence, for both the distance is r. The number of stars counted is then

$$A'(m) = (\omega_1 + \omega_2)\int_0^r r^2 D(r)\varphi(m + 5 - 5\log r)dr + \omega_1\int_r^\infty r^2 D(r)\varphi$$

$$(m - \epsilon_1 + 5 - 5\log r)dr + \omega_2\int_r^\infty r^2\, D(r)\varphi(m - \epsilon_2 + 5 - 5\log r)dr\,,$$

where ω_1 and ω_2 are the solid angles subtended by the two areas. In the analysis we try to represent the numbers $A'(m)$ by an expression of the form

$$A'(m) = (\omega_1 + \omega_2)\int_0^\rho r^2\, D(r)\varphi(m + 5 - 5\log r)dr$$

$$+ (\omega_1 + \omega_2)\int_\rho^\infty r^2\, D(r)\varphi(m - \eta + 5 - 5\log r)dr\,.$$

The problem is to find the relation between the derived absorption η and distance ρ and the true absorptions ϵ_1, ϵ_2 and the true distance r. It is obvious that η is some sort of average between ϵ_1 and ϵ_2; but since we are dealing here with logarithmic quantities, this average is very deceiving. The relation of ρ to r is not at all clear.

The general solution of the problem is difficult and probably of no particular value. We shall, therefore, use a simple illustration[1] to bring out the general weakness of the statistical procedure. Let us assume that, over the range of distances considered, $D(r) = \text{const}$. Let $\varphi(M)$ be given by Van Rhijn's general luminosity function of 1925 for photographic magnitudes. Let $\epsilon_1 = \infty$; $\epsilon_2 = 0$; $\omega_1 = 20\omega_2$; $r = 200$ parsecs. In other words, we assume that we have an area ω_1 where the absorption is infinite, and that in the star counts for the obscured region we inadvertently include a rim having a total area $\omega_2 = \frac{1}{21}\omega_1$, where the apparent stellar distribution is normal. Surely, this case is not exaggerated!

We can now use Bok's numerical method[2] and construct the true curves of $A'(m)$. Figure 1 shows the resulting Wolf diagram.

[1] The following considerations are doubtless known to many astronomers. They are given here because they have been neglected in several practical investigations of dark nebulae.

[2] *Op. cit.*, p. 26.

We see at once that the diagram suggests an absorption of only about 2 mags. Even though the region of complete opacity was twenty times larger than the normal rim, the average η is quite small.

The next step is to interpret the Wolf diagram of Figure 1 by means of Bok's method. A priori we have no knowledge of ρ. But let us suppose we have good luck and we guess $\rho = 200$ parsecs. We then compute a series of curves of $A''(m)$ for $\eta = 1$ mag., $\eta = 2$ mag., ... , $\eta = 5$ mag. These curves, together with the original curves for $A(m)$ and $A'(m)$, are shown in Figure 2*a*. We infer that not one of the curves $A''(m)$ agrees with the curve $A'(m)$. Since the density function $D(r)$ is given by the curve

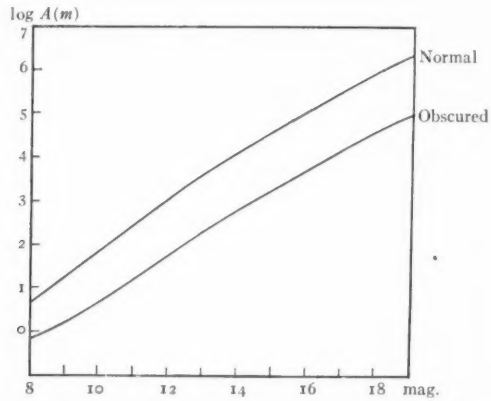


FIG. 1.—Wolf diagram computed for a region in which the absorption is complete over an area of 19/20 of the total. In the remaining 1/20 of the region, the absorption is zero.

$A(m)$, we have no right to vary it, unless we assume that the density of stars depends upon the presence of an absorbing screen. We therefore try to vary ρ. Figures 2*b*, *c*, *d* are drawn for $\rho = 320$ parsecs, $\rho = 500$ parsecs, and $\rho = 800$ parsecs. Figure 2*b* gives a distinctly better fit than Figure 2*a*. Since we are inclined to doubt the lower part of the observed curve because of the small numbers of stars, we shall probably consider as our best fit a value of ρ intermediate between 320 and 500 parsecs, say $\rho = 400$ parsecs. The corresponding $\eta = 2.5$ mag.

We see that we have not only badly misjudged ϵ_1, but that we have not hit the correct value of r. Moreover, even the best fit is not very good. We should probably be inclined to assume that there are two or more nebulae in the line of sight, in order to escape the "sausage-shaped" dark nebulae of former investigators. Our final picture of the dark nebula is

totally wrong! There is little or no use to continue the routine process of gathering this kind of information. Either the statistical processes must be refined or our information must be gathered from individual stars, for which magnitudes, colors, spectral types, and luminosities are known. The latter procedure is tedious, but it promises to give reliable results.

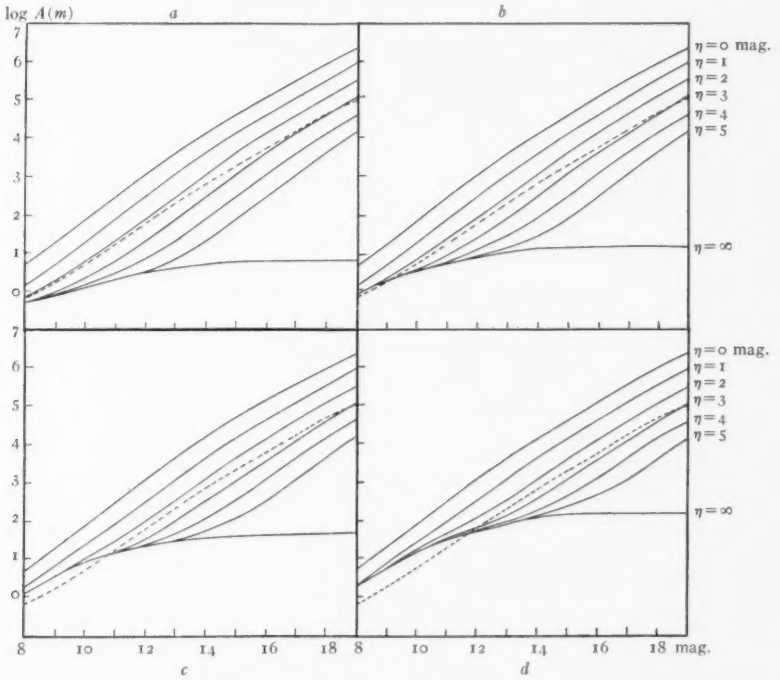


FIG. 2.—Wolf diagram computed for uniform absorptions over the entire region of 1, 2, 3, 4, 5 and ∞ mag., and for distances of (*a*) 200 parsecs, (*b*) 320 parsecs, (*c*) 500 parsecs, and (*d*) 800 parsecs. The dotted curve represents the Wolf diagram of Fig. 1.

Becker's second chapter deals with the general absorption in interstellar space. He stresses the fact that Trumpler had never intended to prove the existence of a uniformly absorbing medium in interstellar space, and concludes that "it has not been possible to prove, by means of an absolute method, the existence of an all pervading interstellar absorption." This view is generally gaining at the present time, and the investigations of Greenstein, of Shapley and his collaborators, and of others support it.

Chapter iii, on the selectivity of interstellar absorption, is interesting because of Becker's own contributions in this field. A rediscussion of the

color excesses of globular clusters by Stebbins and Whitford leads to the surprising result that the reddening toward the galactic plane is quite small. One wonders, in this connection, whether the spectral types used by Becker are sufficiently reliable and whether they are really based upon the absorption lines alone, without consideration of the energy distribution. The relation between spectral type and galactic latitude, first pointed out in this monograph, deserves further investigation.

A short chapter on the optical thickness of the absorbing layer in the galaxy is followed by the concluding chapter on the wave-length dependence of the absorption.

OTTO STRUVE